Extra Credit Session 4 – chapters 1, 2, 3

1 Find the distance between the parallel lines $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + s \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} 2 \\ -2 \end{pmatrix}$.

2 Find the vector equation of the line in $\mathbb{R}^2$ that passes through $P = (2, -1)$ and is parallel to the line $2x - 3y = 1$.

3 Let $\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 2 \\ 5 \\ 7 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} 3 \\ 7 \\ 10 \end{pmatrix}$ and let $A = \begin{pmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{pmatrix}$. Use elementary row operations to obtain a linear dependence relation among $\mathbf{u}, \mathbf{v}, \mathbf{w}$.

4 Consider the linear system

$$\begin{aligned} kx + y + z &= 1 \\ x + ky + z &= 1 \\ x + y + kz &= 1 \end{aligned}$$

For what values of k does the system have a unique solution? No solution? Infinitely many solutions?

5 Consider 4 coins arranged in a single row. A play move consists of flipping two adjacent coins, or just the last coin without flipping any others. Suppose initially the first and third coins are heads and the rest are tails, and the desired final configuration is having all tails except the last coin. Using vectors of binary numbers, model and solve this linear game.
I insist you model this with matrix equation. If you argue by logic, I won't give you credit for this problem.

6 Find the standard matrix representation of the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ which corresponds to reflection in the line $y = x$ followed by projection onto the x-axis.

7 Use LU factorization of $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 7 & 9 \end{pmatrix}$ to solve $A\mathbf{x} = \mathbf{b} = \begin{pmatrix} 5 \\ 4 \\ -1 \end{pmatrix}$.

8 Every year, 10% of all University of Okoboji students change their major to mathematics, and 25% of math majors change their majors to something else, or graduate. If the university enrollment remains a constant 13,500, what is the long term departmental enrollment?

True/False. Give reasons.

9 The rank of a matrix is equal to the number of its nonzero rows.

10 If A is an $m \times n$ matrix of rank m, then the system $A\mathbf{x} = \mathbf{b}$ must have a solution.