Let $\mathbf{v}_1 = [1,1,1]$ and $\mathbf{v}_2 = [1,0,0]$ be vectors in $\mathbb{R}^3$ and let $P$ denote the set of *all* linear combinations of $\mathbf{v}_1$ and $\mathbf{v}_2$. Note: $P$ is a plane. Let $\mathbf{u} \in \mathbb{R}^3$ but $\notin P$. Find a description for $\text{proj}_P(\mathbf{u})$.

In class, we showed that every vector $\mathbf{v} \in P$ must look like

$$\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 = [c_1 + c_2, c_1, c_1]$$

for some $c_1, c_2 \in \mathbb{R}$. Hence

$$P = \{\mathbf{v} | \mathbf{v} = [c_1 + c_2, c_1, c_1] \forall c_1, c_2 \in \mathbb{R}\}.$$

Now, let $\mathbf{u} = [u_1, u_2, u_3]$. Since $\text{proj}_P(\mathbf{u})$ lies in P, there exists $a, b \in \mathbb{R}$ such that $\text{proj}_P(\mathbf{u}) = a\mathbf{v}_1 + b\mathbf{v}_2 = [a + b, a, a]$. If we solve for $a, b$, we will have the description for $\text{proj}_P(\mathbf{u})$ that we are looking for.

Note that $\mathbf{u} - \text{proj}_P(\mathbf{u})$ is orthogonal to $P$ and hence it is also orthogonal to $\mathbf{v}_1, \mathbf{v}_2$. Using this criteria, find the formulas for $a, b$ and write down the description for $\text{proj}_P(\mathbf{u})$.