

Practice Test 2

For full credit, show all work.

1 State the definition for the following:

- a) A basis for a subspace S .
- b) An invertible matrix M .
- c) $\text{rank}(A)$.

2 Given $A = \begin{pmatrix} 3 & 1 & 4 & 6 \\ 2 & 0 & 1 & 3 \\ -1 & 1 & 2 & 0 \end{pmatrix}$. Find the following:

- a) A basis for $\text{null}(A)$.
- b) A basis for $\text{column}(A)$.
- 3a) Show that projection onto $y = 2x$ is a linear transformation.
- b) Find the standard matrix representation for the projection.

4 Find counter-examples to the following false statements: (10 pts each)

- a) $(A + B)^{-1} = A^{-1} + B^{-1}$.
- b) $AB = AC \implies B = C$.

5a) Find the LU factorization for $A = \begin{pmatrix} 2 & 1 & -2 \\ -2 & 3 & -4 \\ 4 & -3 & 0 \end{pmatrix}$.

b) Use the LU factorization to solve the system $A\mathbf{x} = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$.

6 Suppose the weather of a city is a Markov process. The probability that tomorrow is dry is $8/10$ if today is dry, and $4/10$ if today is wet. The probability that tomorrow is wet is $2/10$ if today is dry, and $6/10$ if today is wet.

- a) Write down the transition matrix P for the above Markov process. Show that the matrix is stochastic.
- b) What is the distribution of wet and dry days in the long run?

7 Prove 2 of the following statements (do the remaining one for extra credit):

- a) If A and B are square matrices and AB is invertible, then both A and B are invertible.
- b) If A and B are $n \times n$ matrices of rank n , then AB has rank n .
- c) If R is a matrix in echelon form, then a basis for $\text{row}(A)$ consists of the non-zero rows of R .