Practice Test 2

For full credit, show all work.

1 State the definition for the following:
   a) A basis for a subspace $S$.
   b) An invertible matrix $M$.
   c) rank($A$).

2 Given $A = \begin{pmatrix} 3 & 1 & 4 & 6 \\ 2 & 0 & 1 & 3 \\ -1 & 1 & 2 & 0 \end{pmatrix}$. Find the following:
   a) A basis for null($A$).
   b) A basis for column($A$).

3a) Show that projection onto $y = 2x$ is a linear transformation.
   b) Find the standard matrix representation for the projection.

4 Find counter-examples to the following false statements: (10 pts each)
   a) $(A + B)^{-1} = A^{-1} + B^{-1}$.
   b) $AB = AC \implies B = C$.

5a) Find the LU factorization for $A = \begin{pmatrix} 2 & 1 & -2 \\ -2 & 3 & -4 \\ 4 & -3 & 0 \end{pmatrix}$.
   b) Use the LU factorization to solve the system $Ax = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$.

6 Suppose the weather of a city is a Markov process. The probability that tomorrow is dry is $8/10$ if today is dry, and $4/10$ if today is wet. The probability that tomorrow is wet is $2/10$ if today is dry, and $6/10$ if today is wet.
   a) Write down the transition matrix $P$ for the above Markov process. Show that the matrix is stochastic.
   b) What is the distribution of wet and dry days in the long run?
7 Prove 2 of the following statements (do the remaining one for extra credit):

a) If $A$ and $B$ are square matrices and $AB$ is invertible, then both $A$ and $B$ are invertible.

b) If $A$ and $B$ are $n \times n$ matrices of rank $n$, then $AB$ has rank $n$.

c) If $R$ is a matrix in echelon form, then a basis for row($A$) consists of the non-zero rows of $R$. 