Math 422-522

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Home Work No. 2.

1. Show that Airy functions satisfying equation

$$u_{xx} + xu = 0$$

can be expressed through Bessel functions as follow:

$$u = \sqrt{z}\, J_{\pm 1/3}\left(\frac{2}{3}\, z^{3/2}\right)$$

2. Show that function $M_{0,\mu}(z)$ presented by series

$$M_{0,\mu} = z^{1/2+\mu}\left\{1 + \sum_{n=1}^{\infty} \frac{z^{2k}}{2^{4k}\, k!\, (\mu+1)\cdots(\mu+k)}\right\}$$

is the solution of equation

$$\frac{d^2 M_{0,\mu}}{dz^2} + \left(-\frac{1}{4} + \frac{1/4 - \mu^2}{z^2}\right) M_{0,\mu} = 0$$

3. Find explicit expression for

$$J_{-5/2}\,(x) = ?$$

Next three problems are based on recurrence relations

$$J^{(x)}_{s+1} = -x^s \frac{\partial}{\partial x}\, x^{-s}\, J_s(x)$$

$$J^{(x)}_{s-1} = x^{-s} \frac{\partial}{\partial x}\, x^{s}\, J_s(x)$$

4. Evaluate integral

$$\int_0^\infty J_0(x)\,dx = ?$$

5. Find indefinite integral

$$\int x \ln x \, J_0(x)\,dx = ?$$

6. Show that
$$\int_0^1 x J_0^2(ax)\,dx = \frac{1}{2}\left[J_0^2(a) + J_1^2(a)\right]$$

7. Show that Legendre polynomials are described by the Rodrige's formula

$$P_n(x) = \frac{1}{2^n\,n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n$$

Hint: compare this expression with formula (8) on page 590 of McQuarrie book.