Math 422

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1. Express function $F(\lambda, x) = \sin(\lambda \sin \frac{\pi}{l} x), \quad 0 < x < l$ as a Fourier sine series. λ is a parameter. Hint: use the integral representation for Bessel functions.

2. Express function $f(x) = l^2 - x^2, \quad$ on $\quad 0 < x < l$ as cosine Fourier series. By the use of Parseval's equality show that

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$$

3. Expand function

$$f(x) = \sin \frac{\pi}{l} x$$

in a cosine Fourier series on $0 < x < l$.

4. Obtain expansion

$$\frac{1 - r \cos z}{1 - 2r \cos z + r^2} = 1 + r \cos z + \cdots + r^n \cos z + \cdots$$

5. Solve the Laplace equation $\Delta u = 0$ inside the annocular domain $R_1 < r < R_2$ with boundary conditions

$$u|_{r=R_2} = 0, \qquad \frac{\partial u}{\partial r}\Big|_{r=R_1} = f(\theta)$$

6. Solve the heat equation $u_t = u_{xx}$ on the interval $0 < x < l$ with periodic boundary conditions

$$u(0) = u(l), \quad u_x(0) = u_x(l), \quad u|_{t=0} = u_0(x)$$

7. Solve the equation describing oscillations of a beam with hinged ends

$$u_{tt} + \lambda u_{xxxx} = 0, \quad 0 < x < l$$

$$u(0) = u(l) = 0, \quad u'' = u''(l) = 0$$

$$u|_{t=0} = u_0(x), \quad u_t|_{t=0} = v_0(x)$$

8. Solve the forced string equation

$$u_{tt} = c^2 u_{xx} + f(x,t), \quad 0 < x < l$$

$$u(0) = 0, \quad u_x(l) = 0, \quad u|_{t=0} = 0, \quad u_t|_{t=0} = 0$$

$$f(x) = x(x - 2l)\cos\omega_0 t, \quad \omega_0 = \frac{c\pi}{2l}$$