Math 422
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1. Which functions from this list are harmonic?
    A. $w(x, y) = x^2 - 2y^2$
    B. $w(x, y) = 3x^3 y - y^3$
    C. $w(x, y) = \frac{1}{x^2 + y^2}$
    D. $w(x, y) = \ln(x^2 + y^2)$
    E. $w(x, y) = e^{x-y}$
    F. $w(x, y) = e^x \sin y$

2. Let
$$u = \frac{x^2 - y^2}{(x^2 + y^2)^2}$$

   Find the conjugate function $v(x, y)$, connected with function u by the Cauchy-Riemann conditions:
$$u_x = v_y \qquad u_y = -v_x$$

3. Evaluate the integral
$$\int_{-\infty}^{\infty} \frac{dx}{(1 + x^2)^2}$$

4. Evaluate the integral
$$I(k) = \int_{-\infty}^{\infty} \frac{e^{ikx}}{(1 + x^2)^2} \, dx$$

   Hint: Study the cases $k > 0$ and $k < 0$ separately.

5. Evaluate the integral
$$I = \oint z^4 e^{-1/z} \, dz$$

   Contour of integration is the unite circle.

6. Prove that
$$\frac{\pi^2}{\sin^2 \pi z} = \sum_{n=-\infty}^{n=\infty} \frac{1}{(z - n)^2}$$