Math. 410 — 1. Practice test

Name:

1. Let

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & -1 & -1 & -1 \\ -3 & 0 & 2 & 1 \\ -2 & 1 & 3 & 5 \end{pmatrix}.$$

a) Find the rank of A.
b) Find the dimension of the kernel of A.
c) Find the dimension of the range of A.
d) Find a basis in the range of A.

2. a) For what values of a the phase portrait of the system

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 3 & 3 \\ -4 & a \end{pmatrix} \mathbf{x}$$

is a stable focus?
b) Sketch the phase portrait of the system when $a = -3$.

3. Find an orthogonal basis in the subspace of $\mathbb{R}^4$ spanned by the vectors $(1,1,1,1)^T$, $(1,-1,1,1)^T$, and $(2,-1,2,1)^T$.

4. Find a 2×2-matrix A such that $(1, 2)^T$ is its eigenvector that corresponds to the eigenvalue 2 and $(2, 1)^T$ is its eigenvector that corresponds to the eigenvalue -2.

5. Find the solution of the system

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} \mathbf{x}$$

such that $\mathbf{x}(0) = (1, 1)^T$.

6. Let A_n be the following $n \times n$-matrix

$$A_n = \begin{pmatrix} 1 & 2 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 2 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & 1 & 2 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix}.$$

In words, all diagonal entries of A equal 1, all entries right above the diagonal equal 2, and all other entries equal 0.
a) Find A_4^{-1}.
b) Find A_n^{-1} for an arbitrary n. Explain your answer.