

Math 422-592

Practice test

① A, B are vectors in \mathbb{R}^3

Find

$$(B, Ax(BxA)) = S^2$$

and explain geometric meaning of S .

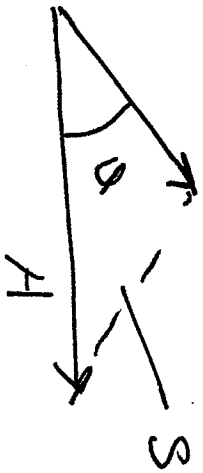
Solution

$$Ax(BxA) = B(AA) - A(BA)$$

$$\begin{aligned} (B, Ax(BxA)) &= B^2 A^2 - (AB)^2 = B^2 A^2 (1 - \cos^2 \theta) = \\ &= A^2 B^2 \sin^2 \theta \end{aligned}$$

$$\text{Here } \cos \theta = \frac{(AB)}{|A||B|}$$

S is the area of the triangle made of vectors \vec{A}, \vec{B}



(2) Solve equation

$Z^3 = 1$
 in cartesian and in exponential representation
 of complex numbers

Solution:

$$Z_1 = 1$$

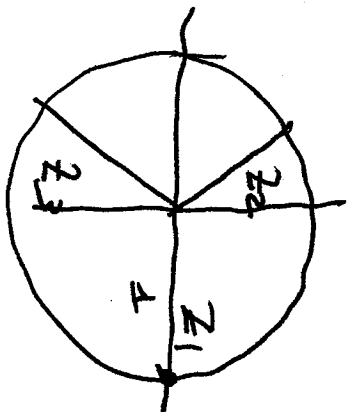
$$Z_2 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$Z_3 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$$

$$Z_1 = 1$$

$$Z_2 = 2$$

$$Z_3 = 2$$



③ Determine the radius of convergence of the following series

$$S(z) = \sum_{n=1}^{\infty} z^n$$

Solution

$$\lim_{n \rightarrow \infty} \left(\frac{z^{n+1}}{z^n} \right)^{1/n} = \lim_{n \rightarrow \infty} e^{\frac{\ln |z|}{n}} = 1$$

$$R = 1$$

④ Find the second solution of the Legendre equation at $n=1$

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0$$

-4-

Solution. See page 591

$$Q_1(x) = \frac{x}{2} \ln \frac{1+x}{1-x} - 1$$

⑤ Show that $\int y_0(x) y_1(x) dx = -\frac{1}{2} y_0^2(x)$

In lead

$$y_1 = -\frac{2y_0}{2x}$$

$$y_0 y_1 = -y_0 \frac{2y_0}{2x} = -\frac{1}{2} \frac{d}{dx} y_0^2(x)$$