COMPUTER SIMULATION OF THE LANGMUIR COLLAPSE

S.I. ANISIMOV, M.A. BEREZOVSKIĬ, M.F. IVANOV, I.V. PETROV, A.M. RUBENCHICK and V.E. ZAKHAROV

L.D. Landau Institute for Theoretical Physics, The Academy of Sciences of the USSR, Moscow, USSR

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Computer simulation of the collapse of Langmuir waves is carried out, using the PIC method. The high-frequency electric field energy has been shown to transform almost completely into electron energy at the final stage of the collapse.

The phenomenon of collapse of Langmuir waves [1] consists of the spontaneous appearance of low density regions (cavities) in a plasma filled with confined electron plasma oscillations. After a finite time the cavities are constricted, transmitting the oscillation energy to the electrons. The collapse of Langmuir waves provides an effective mechanism of dissipation for long wavelength ($kr_D \ll 1$), high amplitude plasma waves. It possibly plays the most important role in the physics of Langmuir wave turbulence. The small size of the cavities ($\leqslant 100 r_D$) and their short lifetimes ($\leqslant 10^3 \omega_{pe}^{-1}$) make the experimental observation of this important phenomenon difficult. The information available concerning the collapse results from scaling considerations, as well as from numerical solution of the time-averaged dynamical equations which describe the evolution of the envelope of the amplitude of the high-frequency field [1–3]. This last approach was used to study the evolution of a single cavity [2–6] and the generation of "tails" in the electron distribution function [2,7–9]. However, the averaged equations fail at the end phase of the collapse when dissipation of the energy of the Langmuir oscillations takes place. For a complete description of the phenomenon it is necessary to solve the set of Vlasov's equations for electrons and ions. For this reason numerical simulation of the collapse is also of fundamental importance, since it is an "ab initio" calculation and thus provides a full alternative to a laboratory experiment. At present the only approach to this problem is the computer simulation of the plasma by the PIC-method. This method presents a number of difficulties which may explain why an exact kinetic solution of the basic problem of the evolution of a single cavity has not been obtained. The attempts made [10,11] have given results which can hardly be interpreted in terms of collapse.

This work presents results of a two-dimensional simulation of the problem. It is shown that under properly chosen initial conditions one can reproduce the picture of collapse resulting from the solution of averaged dynamical equations [2–6]. During the collapse the electric field energy density inside the cavity increases by about one order of magnitude. After this the oscillation energy is almost completely transferred to fast electrons.

The difficulties encountered in computer simulation of collapse are due to the discreteness of the model. To obtain a wide enough inertial range (i.e., a large enough ratio of final to initial energy density W) one should start the simulation with a small W compatible with the collapse condition [1]:

$$W/nT > (kr_D)^2 , \qquad (1)$$

where k is a characteristic wave-number. The value kr_D cannot be taken appreciably less than 0.1 due to limited resources of existing computers. One more limitation arises from the fact that the initial electrostatic energy density must be much higher than the level of thermal fluctuations (which is inversely pro-

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portional to the number of particles in a Debye cell). To fulfill both requirements one must perform the simulation with a very large number of particles, which leads to extremely long computations.

With a limited number of particles the proper choice of initial conditions becomes very essential. The problem is solved in a square domain $-L < x < L$, $-L < y < L$ with $L = 64 r_D$. The initial macroscopic velocities of electrons and ions are taken to be zero. The initial densities of electrons and ions are chosen as follows:

$$n_e = n_0 + \delta n + \delta \bar{n}\,, \quad n_i = n_0 + \delta n\,,$$

$$\delta \bar{n}/n_0 = \epsilon \cos(\pi x/L)\cos(\pi y/L)\,,$$

$$\delta n/n_0 = \epsilon_1 \cos(2\pi x/L)\cos(2\pi y/L)\,.$$

The parameter ϵ_1 is chosen such that the initial high-frequency electric field

$$E = \tfrac{1}{2}\,[(\nabla\psi)\exp(i\omega_{pe}t) + \text{c.c.}]$$

(which is dependent on the value of δn) satisfies the equation

$$\langle |E|^2\rangle/16\pi n_0 T = -\delta n/n_0\,. \qquad (2)$$

It means that the thermal and the high-frequency electrostatic pressure are in equilibrium in the cavity at the initial moment. According to refs. [2–5], condition (2) should hold during the time of the cavity constriction.

The criterion of collapse in a 2D-case is $I < 0$, I being

$$I = \int \mathrm{d}x\,\mathrm{d}y \left(\frac{3\omega_{pe} r_D^2}{64\pi} |\Delta\psi|^2 + \frac{\omega_{pe}\delta n}{16\pi n_0} |\nabla\psi|^2 + \frac{c_s^2(\delta n)^2}{2n_0} + \frac{M n_0 v^2}{2} \right). \qquad (3)$$

By substituting the above expressions into (3) the critical condition for the collapse reduces to $\epsilon > \epsilon_c \approx 0.008$. Note that in the simulations made in refs. [10,11] the condition that the integral (3) be negative was not fulfilled.

The simulation was carried out using a multiprocessor M-10. The code employed was described in ref. [12]. It is similar to the code worked out by Dawson et al. [13]. A major part of the results was obtained with about 10^5 particles (25 particles of each sort per Debye cell). Ion-to-electron mass

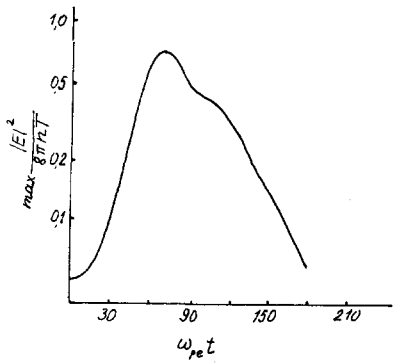

Fig. 1. Normalized electrical field energy density at the center of the cavity as a function of time.

ratio varied in different runs from 100 to 800. Simulation with $\epsilon = 0.01$ gave no definite picture of the collapse (field distribution was similar to that obtained in ref. [10]). At $\epsilon = 0.025$ and 0.05 a clear picture of the collapse of a single cavity was observed. The time of collapse decreased with increasing ϵ. The contours of constant energy density of the electric field are qualitatively similar to those obtained in ref. [2]. In particular, the cavity is also nonsymmetric as in ref. [2]. During the collapse the field energy inside the cavity increases by a factor ≈ 16 at $\epsilon = 0.05$ (see fig. 1). After this the damping is "switched on" sharply and almost all the field energy localized inside the cavity is converted into energy of fast electrons. The distribution function of electrons at $\omega_{pe}t = 170$ is shown in fig. 2.

A detailed consideration of the results of the present simulation confirms the general picture of cavity collapse and wave energy dissipation given in refs. [1–9]. In particular, the major part of the wave

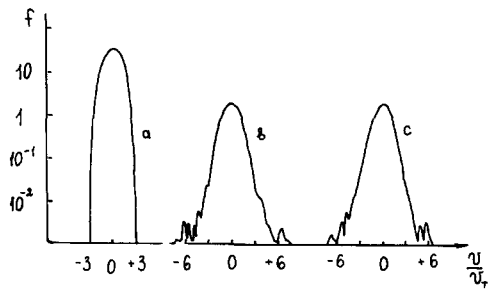

Fig. 2. The electron distribution function: (a) maxwellian distribution at $t = 0$, (b) $f(v_x)$ at $\omega_{pe}t = 170$, (c) $f(v_y)$ at $\omega_{pe}t = 170$.

energy confined inside the cavity is dissipated at the final stage of the collapse.

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