Interesting Problems
Mathematics 425A: Analysis
August 28, 2006

1. Write \( \frac{1}{717} \) as a repeating decimal.

2. Suppose \( c > 0 \) is a real number that can be rapidly approximated by rational numbers \( s_n \). This means that \( c = s_n + r_n \) where for each \( n \) the number \( s_n \) is rational, \( n!s_n \) is integer, the remainder \( r_n > 0 \), and where \( \lim_{n \to \infty} n!r_n = 0 \). Prove that \( c \) is irrational.

3. Let \( s_n, n = 1, 2, 3, \ldots \) be a sequence of real numbers. Each strictly increasing sequence \( N_k, k = 1, 2, 3, \ldots \) of natural numbers defines a corresponding subsequence \( s_{N_k}, k = 1, 2, 3 \) of real numbers.

Is there a sequence \( s_n, n = 1, 2, 3, \ldots \) of real numbers such that for every real number \( y \) there is a subsequence \( s_{N_k}, k = 1, 2, 3, \ldots \) that converges to \( y \)? Either give an example, or prove that no such example exists.