1. Compute the following integral:

\[ \int_0^\infty \frac{\cos x}{x^2 + 1} \, dx \]

2. Find a conformal mapping of the strip \( 0 < \text{Re}(z) < 1 \) onto the unit disc \( |w| < 1 \).

3. Prove, directly from the definition of a covering space, that if \( p : Y \to X \) is a covering map between topological spaces then every continuous map \( \phi : [0,1] \to X \) lifts to a continuous map \( \psi : [0,1] \to Y \) such that \( p \circ \psi = \phi \).

4. Compute the De Rham cohomology groups of the connected sum \( \mathbb{RP}^3 \# \mathbb{RP}^3 \).

5. Let \( V \) be the following vector field on \( \mathbb{R}^2 \):

\[ V = y \frac{\partial}{\partial x} + x \frac{\partial}{\partial y}. \]

(a) Compute and graph the flow of \( V \).

(b) Determine whether the flow preserves the area form \( dx \wedge dy \).

6. Does there exist a differentiable map \( F : T^4 \to T^4 \) of the four-dimensional torus \( T^4 = \mathbb{R}^4 / \mathbb{Z}^4 \) into itself such that \( F^*[dx_1 \wedge dx_2] = [dx_2 \wedge dx_3] \) and \( F^*[dx_1 \wedge dx_3] = [dx_1 \wedge dx_4] \), where \([\cdot]\) stands for the de Rham cohomology class and \( F^* \) is the map induced by \( F \) in cohomology?

7. Assume that a 6-element group \( \Gamma \), isomorphic to the group of permutations in three letters, acts on \( X = S^3 \) freely. Compute \( \pi_1(Y) \) and \( H_1(Y, \mathbb{Z}) \), where \( Y = X/\Gamma \).

8. Prove the Poincaré Lemma in the plane: if \( \alpha \) is a closed \( k \)-form on \( \mathbb{R}^2 \) and \( k > 0 \), then \( \alpha \) is exact.