Section 3.3 – Graphs of Basic Functions; Piecewise Functions

Objectives

- Identify by sight the graphs of basic functions, such as the constant function, the identity function, the square function, the cube function, the absolute value function, the square root function, the cube root function, the reciprocal function, and the greatest integer function.
- Evaluate a piecewise-defined function at a value in its domain.
- Graph a piecewise-defined function.
- Given the graph of a piecewise-defined function, find a rule that describes the function.
- Solve application problems involving piecewise-defined functions.

Preliminaries

Sketch the graph of each of the following functions. Indicate domain, range, and intercepts.

\[ y = x \quad y = x^2 \quad y = x^3 \]

\[ y = \sqrt{x} \quad y = \frac{1}{\sqrt{x}} \quad y = |x| \]
Warm-up

6. Find the intercepts algebraically and graph the function: \( g(x) = -2x + 5 \).

7. Find the equation of the line passing through the points \((-3, -2) \) and \((-1, 5)\).
Class Notes and Examples

What is meant by a **piecewise-defined function**?

How do you **evaluate** a piecewise-defined function?

How do you **graph** a piecewise-defined function?
3.3.1 For the function \( f(x) = \begin{cases} x - 1 & x \leq -2 \\ 2x + 3 & x > -2 \end{cases} \)

(A) Sketch an accurate graph of \( f \).

(B) What is/are the intercept(s) of \( f \)?
3.3.2 Consider the piecewise function \( f(x) = \begin{cases} x^2 & x < 0 \\ 2x - 1 & 0 \leq x \leq 1 \\ \sqrt{x} & x > 1 \end{cases} \)

(A) Evaluate \( f(-3) \), \( f(1) \), and \( f(5) \).

(B) Sketch an accurate graph of \( f \).

(C) What are the intercepts of \( f \)?

(D) What is the domain of \( f \)?

(E) What is the range of \( f \)?
3.3.3 Give a rule for the function graphed below.
3.3.4 Give a rule for the function graphed below.
3.3.5 A small business is planning on ordering boxes of printing paper for the upcoming year.

The quoted price for ordering the paper is as follows:
- The set-up fee for the order is $20.
- The first 80 boxes of paper are priced at $50 each.
- Any additional boxes of paper are priced at $45 each.

(A) Find a formula for the piecewise-defined function that represents the total cost of ordering $n$ boxes of paper.

(B) How much would it cost to order 200 boxes of paper?
1. Use the function \( g(x) = \begin{cases} x^2 + 1 & x < 3 \\ 2x - 8 & x \geq 3 \end{cases} \) to answer the following questions.

   (A) Evaluate \( g(2) \).

   (B) Determine all of the intercepts of \( g(x) \).

2. Sketch an accurate graph of the function \( f(x) = \begin{cases} x + 3 & x < 1 \\ -2x + 4 & x \geq 1 \end{cases} \).
3. **(Multiple Choice)** Determine the rule that defines the piecewise-defined function graphed below.

\[ f(x) = \begin{cases} 
(x - 2)^2 & x < 1 \\
-2x + 4 & x \geq 1 
\end{cases} \]

(A) \( f(x) = \begin{cases} 
(x - 2)^2 & x < 1 \\
-2x + 4 & x \geq 1 
\end{cases} \)

(B) \( f(x) = \begin{cases} 
x^2 - 2 & x < 1 \\
-2x + 6 & x \geq 1 
\end{cases} \)

(C) \( f(x) = \begin{cases} 
(x - 2)^2 & x \leq 1 \\
-2x + 4 & x > 1 
\end{cases} \)

(D) \( f(x) = \begin{cases} 
x^2 - 2 & x \leq 1 \\
-2x + 6 & x > 1 
\end{cases} \)

(E) None of these