Section 4.2 – Applications and Modeling of Quadratic Functions

Objectives
- Set up a quadratic function to model certain "real life" situations.
- Use quadratic models and interpret results in "real life" terms.
- Solve application problems involving quadratic functions generated from projectile motion models, area models, and demand/ cost/ revenue/profit models.

Preliminaries
Summarize key formulas from this section:

**Area formulas** for the following shapes

- Rectangle
- Circle

**Surface area formulas**

- Rectangular box
- Cylinder

**Business model definitions**

The demand function models ______________ as a function of ______________.

The revenue from selling \( x \) items at a price \( p \) is given by the formula ____________.
Warm-up

1. A rectangle has a perimeter of 10 inches. Express the area of the rectangle as a function of its width.

2. Suppose the fixed cost of producing a line of sunglasses is $25,000 and each pair of sunglasses costs $3 to make. Find a formula for the total cost of producing $s$ pairs of sunglasses.
Class Notes and Examples

When a physical situation can be described by a quadratic function, what feature of the graph will give you the optimal (maximum or minimum) value?

How do you know whether the vertex of a quadratic function is a maximum or a minimum?

If the zeros of a quadratic function occur at \( x = r_1 \) and \( x = r_2 \), at what value of \( x \) will the vertex occur?

What point gives the vertex for a quadratic function of the form \( y = a(x - h)^2 + k \)?
4.2.1 A stone is thrown upward; its height in meters $t$ seconds after release is given by
\[ h(t) = -4.9t^2 + 49t + 277.4. \]

(A) How high was the stone when it was released?

(B) How long will it take the stone to hit the ground? (Round to the nearest 0.01 second)

(C) When will the stone be at its maximum height?

(D) What is the maximum height? Verify by graphing.
4.2.2  A rancher has 12,000 feet of fencing and wants to enclose a rectangular field with an internal fence parallel to one side, as shown below. What is the maximum total area that can be enclosed?
4.2.3 Suppose a sunglass manufacturer determines the demand function for a certain line of sunglasses is given by \( p = 50 - \frac{1}{4,000}x \), where \( p \) is the price per pair and \( x \) is the number of pairs sold. The fixed cost of producing a line of sunglasses is $25,000 and each pair of sunglasses costs $3 to make.

(A) Express the total cost as a function of \( x \).

(B) Express the revenue as a function of \( x \).

(C) Express the profit as a function of \( x \).

(D) How many sunglasses should be produced to maximize profit?
4.2.4 A concert venue holds a maximum of 1000 people. With ticket prices at $30, the average attendance is 650 people. It is predicted that for each dollar the ticket price is lowered, approximately 25 more people attend.

(A) Create a function to represent the revenue generated from ticket sales. (You will need to decide what your variables should represent. Creating a table of values may help you get started.)

(B) What is the maximum possible revenue from this concert? How should the tickets be priced and how many people will attend at that price?
Section 4.2 Self-Assessment (Answers on page 256)

1. (Multiple Choice) Bob’s high school is performing a musical. If each ticket is priced at $12 then 800 tickets will be sold. For each $0.25 increase in price, 20 fewer tickets will be sold. Determine the maximum possible revenue from the musical ticket sales.

   (A) More than $10,000
   (B) Between $9900 and $10,000
   (C) Between $9800 and $9900
   (D) Between $9700 and $9800
   (E) Less than $9700

2. A stone is thrown upward. Its height in meters $t$ seconds after release is given by

   $$ h(t) = -4.9t^2 + 58.8t + 3.7. $$

   (A) How long will it take the stone to hit the ground? (Round to the nearest 0.01 seconds)

   (B) What is the maximum height of the stone?
3. (Multiple Choice) A company has determined that the demand function for a 96-inch couch is given by \( p = 2700 - 0.75x \), where \( p \) is the price per couch and \( x \) is the number of couches sold. The fixed cost of producing a line of couches is $760,000 and each couch costs $360 to make. Determine how many couches should be manufactured and sold to maximize profit.

(A) Less than 1400 couches  
(B) Between 1400 and 1460 couches  
(C) Between 1460 and 1520 couches  
(D) Between 1520 and 1580 couches  
(E) More than 1580 couches


4. A rancher has 4800 feet of fencing and wants to enclose a rectangular field with an internal fence parallel to one side, as shown below. What is the maximum total area that can be enclosed?

\[
\text{L} \quad \text{W} \quad \text{W} \quad \text{W} \quad \text{L}
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