Section 5.3 – Logarithmic Functions

Objectives

- Apply the definition of a logarithm to convert an equation in logarithmic form into exponential form.
- Given an equation in exponential form, rewrite the equation in logarithmic form.
- Properly identify the notation for natural (base e) and common (base 10) logarithm.
- Evaluate natural and common logarithms on a calculator.
- Know and identify the shape and basic features of the graph of \( f(x) = \ln x \) and \( f(x) = \log x \).
- Evaluate simple logarithms of any allowable base without the use of a calculator.
- Determine the domain of logarithmic functions.

Preliminaries

Define in your own words what it means for a function to be the inverse of another function. (see section 3.6 if you need a little review.)

If (1,2) is on the graph of \( y = f(x) \), what point must be on the graph of \( y = f^{-1}(x) \)? (see section 3.6 if you need a little review.)

Rewrite the equation in exponential form.

\[ y = \log_b x \] is equivalent to ________________________________

Rewrite the equation in logarithmic form.

\[ y = b^x \] is equivalent to ________________________________

Write the cancellation properties of exponentials and logarithms.

__________________________________________________________
__________________________________________________________
Warm-up

1. Consider the function $f(x) = 2^x$. Answer the following questions

(A) Sketch an accurate graph of $y = f(x)$. Label coordinates for two points.

(B) State the domain of $f$.

(C) State the range of $f$.

(D) State the asymptote(s), if any.

(E) Explain why $f(x) = 2^x$ has an inverse function.

Sketch an accurate graph of $y = f^{-1}(x)$.

Label coordinates for two points.

State the domain of $f^{-1}$.

State the range of $f^{-1}$.

State the asymptote(s), if any.
Definition: For $x > 0$, $b > 0$, and $b \neq 1$, the logarithmic function with base $b$ is defined by $y = \log_b x$ if and only if $x = b^y$.

5.3.1 Change each exponential equation into logarithmic form.
(A) $2^3 = 8$  (B) $3^{-2} = \frac{1}{9}$  (C) $16^{1/2} = 4$

5.3.2 Change each logarithmic equation into exponential form.
(A) $\log_2 32 = 5$  (B) $\log_{1/3} 27 = -3$

5.3.3 Evaluate each expression without using a calculator. Verify your answer by evaluating the equivalent exponential expression.
(A) $\log_3 9$  (B) $\log_2 \frac{1}{8}$  (C) $\log_4 2$
Notation:

**Common logarithm**: \( \log_{10}(x) \) is denoted simply as \( \log(x) \)

**Natural logarithm**: \( \log_{e}(x) \) is denoted simply as \( \ln(x) \)

Useful for approximating values on a calculator.

7 What are the **basic properties** of logarithms? (Assume \( b > 0 \) and \( b \neq 1 \).)

\[
\text{log}_b b = \underline{\quad} \\
\text{log}_b 1 = \underline{\quad} \\
b^{\text{log}_b x} = \underline{\quad} \\
\text{log}_b (b^x) = \underline{\quad}
\]

5.3.4 Use the basic properties of logarithms to simplify the following expressions without using your calculator.

(A) \( \log_3(3^{12}) \)

(B) \( \log \sqrt{1000} \)

(C) \( 5^{\log_5(11)} \)
Consider the function \( g(x) = \log_2 x \). Note that \( g(x) \) is the inverse of the function \( f(x) = 2^x \) described in the warm-up section. Use the properties of inverse functions and the warm-up exercise to answer the following.

(A) Sketch an accurate graph of \( g(x) \), labeling coordinates for two points.

(B) State the domain of \( g(x) \).

(C) State the range of \( g(x) \).

(D) State the asymptote(s) of \( g(x) \), if any.
5.3.6  Summarize the properties of the graphs of logarithmic functions with different bases.

\[ y = \log_b x \quad \text{for } b > 1 \]

\[ y = \log_b x \quad \text{for } 0 < b < 1 \]

<table>
<thead>
<tr>
<th>Domain:</th>
<th>Domain:</th>
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<tbody>
<tr>
<td>Range:</td>
<td>Range:</td>
</tr>
<tr>
<td>x-intercept:</td>
<td>x-intercept:</td>
</tr>
<tr>
<td>Asymptote:</td>
<td>Asymptote:</td>
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<tr>
<td>Increasing or Decreasing?</td>
<td>Increasing or Decreasing?</td>
</tr>
<tr>
<td>Graph:</td>
<td>Graph:</td>
</tr>
</tbody>
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How do we determine the domain of logarithms?

5.3.7 Find the domain of each logarithmic function.
(A) \( f(x) = \log(3x - 2) \)

(B) \( K(x) = \log_5(4 + 7x) \)

(C) \( M(x) = \ln(x^2 - 4) \)
5.3.8 For each logarithm function below, determine the domain, intercepts, and the equation of the asymptote. Then sketch the graph of each function, labeling the exact coordinates of at least two points.

(A) \( T(x) = \ln(x - 1) + 3 \)
(b) \[ B(x) = \log_4(3x + 2) \]
Section 5.3 Self-Assessment (Answers on page 257)

1. **(Multiple Choice)** Write in logarithmic form.
   
   \[ 26^C = 3 \]
   
   (A) \( \log_C(26) = 3 \) \hspace{2cm} (B) \( \log_3(26) = C \) \hspace{2cm} (C) \( \log_C(3) = 26 \)
   
   (D) \( \log_{26}(3) = C \) \hspace{2cm} (E) \( \log_{26}(C) = 3 \)

2. **(Multiple Choice)** Write in exponential form.
   
   \[ \log_B(6) = 37 \]
   
   (A) \( B^6 = 37 \) \hspace{2cm} (B) \( 37^6 = B \) \hspace{2cm} (C) \( 37^B = 6 \)
   
   (D) \( 6^B = 37 \) \hspace{2cm} (E) \( B^{37} = 6 \)

3. Determine the domain of the logarithmic function \( g(x) = \log_3(8x + 5) \).

4. Determine the \( x \)-intercept(s), if any, of \( f(x) = \log_4(3x + 11) \).