MATH 122A
FINAL EXAM STUDY GUIDE
(Fall 2015-Spring 2016)

The format of the questions on the final exam is multiple choice. The questions in this study guide are not multiple-choice in order to encourage you to solve the problems completely. These are not samples of questions that will appear on the final, but they do provide practice for the material that will be covered. Answers will be provided in a separate file.

1. Find the domain of the following functions:
   a) \( g(t) = \sqrt{t^2 - 20} \)
   b) \( h(y) = \frac{y+5}{\sqrt[3]{y-2}} \)

2. Gasoline is being pumped into a tank at a constant rate (cubic feet per minute). For each of the tanks below, sketch a graph of the height of the water in the tank as a function of time. You can assume the tank is initially empty and will be filled. Also assume the shape and orientation of the tank is as shown.

   a)
   
   b)
   
   c)

3. The lift \( L \) on an airplane wing at take-off is proportional to the square of the speed \( s \) of the plane and the area \( A \) of its wings.

   a) Write an equation for lift.

   b) If the speed is only half as much, how much larger should the area of the wings be for the lift to be the same?

4. Suppose the target heart rate when exercising, \( R \), is a function of a person’s age, \( A \), and is given by the linear formula \( R = H_0 + p(220 - A - H_0) \). Assume \( H_0 \) and \( p \) are constants. Find the slope and give a practical interpretation.

5. For certain category hurricanes, the cube of the diameter \( D \) (in miles) of the hurricane is roughly proportional to the square of the hurricane’s duration \( t \) (in hours).

   a) Write an equation to represent the relationship between diameter of the hurricane and duration. Solve your equation for \( D \).

   b) If the diameter of hurricane Veronica was 302 miles with a duration of 273 hours, what would be the diameter of a hurricane with a duration of 140 hours according to this model?
6. From the given data, determine if Account Balance is a function of Week.

<table>
<thead>
<tr>
<th>Account Balance</th>
<th>$3,250</th>
<th>$3,500</th>
<th>$3,450</th>
<th>$3,700</th>
<th>$3,500</th>
<th>$3,800</th>
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<tbody>
<tr>
<td>Week</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

7. Let \( p(t) = \frac{5}{t + 1} \). Find \( \frac{p(t + h) - p(t)}{h} \) and simplify as much as possible.

8. Suppose an oil spill covers a circular area and that the radius increases according to \( r(t) = 4 + \sqrt{\frac{t}{2}} \) where \( t \) represents the number of minutes since the spill was first observed. The radius is measured in inches.
   a) What was the radius of the spill when it was first observed?
   b) Express the area \( A \) of the oil spill as a function of \( t \).
   c) Find the exact time when the area of the spill is \( 81\pi \) square inches.

9. Use the graph of \( g(x) \) given at the right to answer the questions.

   a) If \( f(x) = x^3 + 2 \), find \( g(f(1)) \).
   b) Find the range of \( g^{-1}(x) \).
   c) Is \( y = |g(x)| \) a one-to-one function?
   d) Find \( \frac{g(4) - g(2)}{4 - 2} \). What does this number represent on the graph of \( g(x) \)?
   e) For what value(s), if any, would \( y = \frac{1}{g(x) - 20} \) be undefined on the interval \([0,5] \)?
   f) Does \( g(x) \) appear to be concave up or concave down on the interval \( 2.5 \leq x \leq 3.5 \)?

10. For functions \( f(t) = 5t^3 - 1 \) and \( g(t) = \sqrt{t + 1} \), find and simplify:
   a) \( e^{f(t)+1} \)  
   b) \( f(t) + (g(t))^2 \)  
   c) \( g(t^2) \)  
   d) \( f(2t) \)
11. One of the functions below is linear, one is exponential, and one is quadratic. Determine which is which and then find a formula for each.

<table>
<thead>
<tr>
<th>Table A</th>
<th>Table B</th>
<th>Table C</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>f(x)</td>
<td>x</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.8</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>7.2</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>16.2</td>
<td>6</td>
</tr>
<tr>
<td>12</td>
<td>28.8</td>
<td>8</td>
</tr>
</tbody>
</table>

12. Sketch a graph of \( y = f(x) \) that satisfies all of the following conditions:
   * domain \(-2 < x < 2\)
   * \( f(x) \) is an odd function
   * range \(-\infty < y < +\infty\)
   * \( f(x) \) is not invertible

13. Let \( f(x) = Ax + B \) and \( g(x) = Cx + D \) where \( A \neq 0, \ C \neq 0 \). Find \( h(x) = (g \circ f)^{-1}(x) \).
What are the slope and vertical intercept of \( h(x) \)?

14. If the zeros of \( g(x) \) are \( x = -2 \) and \( x = 3 \), what are the zeros of \( y = 2g(x+1) \)?

15. Let \( f(x) = \frac{3}{x} \) and \( g(x) = \frac{4}{x^2} - 1 \). Find \( f \left( g(x) \right) \) and simplify your answer.

16. Let \((2, -5)\) be a point on the graph of \( y = f(x) \). Find the corresponding point on the graphs of each of these transformations.
   a) \( y = -f(x) + 4 \)
   b) \( y = f \left( \frac{1}{4}x \right) \)
   c) \( y = 3f(-x) \)

17. It is predicted that the population of a particular state will double every 25 years.
   a) Determine the annual and monthly growth rates. Express your answers as percents.
   b) Determine the continuous growth rate per year. Express your answer as a percent.
18. A typical cup of coffee contains about 100 mg of caffeine. Every hour approximately 16% of the amount of caffeine in the body is metabolized and eliminated (decays).

a) Write an equation for the amount of caffeine, \( C \), in the body as a function of \( t \), the number of hours since a single cup of coffee was consumed.

b) Find the time when 20% of the caffeine has been metabolized and eliminated from the body. Give both an exact answer and a decimal answer. Include units.

19. In 1997, the average tuition at four-year public universities was $2,360 per year. In 1998, that figure rose to $2,430 per year.

a) If tuition increased linearly, write a formula for the tuition as a function of years since 1997. Use your function to estimate the tuition in 2006.

b) If tuition increased exponentially, write a formula for the tuition as a function of years since 1997. Use your function to estimate the tuition in 2006. Note: The average tuition in 2006 was actually $4,102.

20. Let \( f(x) = 3\log_2(8 - 7x) - 12 \)

a) Find the domain and range.

b) Use algebra to find the intercepts of \( f(x) \). Simplify your answers.

21. Fish are introduced into a large lake system. The population size (in numbers of fish) can be modeled by \( P(t) = 2000 - 500e^{-0.03t} \) where \( t \) is measured in months since the fish were introduced.

a) Find \( P(3) \) and give a practical interpretation.

b) Find \( P^{-1}(1500) \) and give a practical interpretation.

c) Is the population of the fish increasing or decreasing?

d) When does the population size reach 1000 fish according to the model?

e) What happens to the population size as \( t \to \infty \)?
22. a) Expand the following expression completely and simplify: \( \log_e \left( \sqrt[4]{\frac{x^2y}{a^3}} \right) \)

b) Combine the following expression into a single logarithm:
\[
\ln(x+1) - 3\ln(x) + \frac{2}{3}\ln(x^2+1)
\]

23. The graph of \( y = \ln x \) along with a line passing through intersecting points A and B is shown below.

   a) Find the coordinates of A and B.
   b) Find the equation of the line.

24. Answer true or false:
   a) _________ The domain of all polynomials is \((-\infty, \infty)\).
   b) _________ If \( g(x) \) passes the vertical line test, it is a one-to-one function.
   c) _________ An even degree polynomial must have at least one maximum or minimum.
   d) _________ All rational functions have vertical asymptotes.
   e) _________ If \( y = f(x) \) is an odd function, then \( y = |f(x)| \) is an even function.

25. Answer the following questions about the polynomial graphed below.
   a) What is the smallest possible degree?
   b) Is the leading coefficient positive or negative?
   c) Write a possible equation for this polynomial.
26. A cable must be laid from point A to a point C across a river. The plan is to go from point A to point B under water and then continue from point B to point C on land. The cost of cable laid under water is $74 per foot while the cable laid on land is $48 per foot. Write an equation for the total cost of laying the cable in terms of \( d \).

27. Find all intercepts, asymptotes, and holes (if any) for \( f(x) = \frac{3x^3 + 14x - 5}{(x+5)(5-4x)} \).

28. In each case, find the value(s) of \( k \) so that the following is true for \( p(t) = \frac{2t^2 + k}{3t+1} \).
   a) \( p(1) = 5 \)  
   b) \( p(3) = 0 \)  
   c) The graph of \( p(t) \) has no zeros.

29. Each time a person’s heart beats, their blood pressure increases and then decreases as the heart rests between beats. A certain person’s blood pressure is modeled by the function \( b(t) = A + B \sin(Ct) \) where \( b(t) \) is measured in mmHg and \( t \) is measured in minutes. Find values for \( A, B, \) and \( C \) if the person’s average blood pressure is 115 mmHg, the range in blood pressure is 50 mmHg, and one cycle is completed every 1/80 of a minute.

30. The Columbia Tower in Seattle is 954 feet tall. The Seafirst Tower is \( T \) feet tall and stands \( d \) feet away from the Columbia Tower. Find the height of the Seafirst Tower. Give your answer to 2 decimal places.
31. If \( \csc(\theta) = \frac{3}{x} \), express \( \tan(\theta) \) in terms of \( x \).

32. Let \( f(\alpha) = \cos(\alpha) \) and \( g(\alpha) = \frac{\pi}{4} \sin(\alpha) \). Find the exact value of \( f \left( g \left( \frac{3\pi}{2} \right) \right) \).

33. Substitute \( x = 3 \sec \theta \) into the expression \( \frac{\sqrt{x^2 - 9}}{x} \) and simplify as much as possible. Assume \( 0 < \theta < \frac{\pi}{2} \).

34. Suppose \( \sin \theta = A \) for \( \frac{\pi}{2} < \theta < \pi \).
   a) What are the possible values of \( A \)?
   b) Find each in terms of \( A \): \( \sin^3 \theta \) \( \cot \theta \) \( \sin \left( \frac{\pi}{2} - \theta \right) \)

35. On a day when the sun passes directly overhead at noon, a six foot tall man casts a shadow of length \( L(t) = 6 \cot \left( \frac{\pi}{12} t \right) \) where \( L \) is in feet and \( t \) is the number of hours since 6 a.m.
   a) Find exact values for the lengths of the shadow at 8:00 a.m., noon, and 2:00 p.m.
   b) Use your calculator to help you sketch an accurate graph of \( L(t) \) for \( 0 < t < 12 \).
   c) Determine the values of \( t \) at which the length of the shadow equals the man’s height.

36. Find a possible formula for the functions graphed below.
   a) Use the general exponential form.
   b) Use a periodic function.
c) The vertical asymptote is \( x = -3 \) and the horizontal asymptote is \( y = -1 \).

37. In each problem solve for the indicated variable on the given interval. Do not use your calculator.

a) For \( t \): \( \cos^2(t)\sin(t) + \sin(t) = 0 \quad 0 \leq t \leq 2\pi \)

b) For \( x \): \( \frac{\cos\left(\frac{x}{2}\right)}{2 - \sin\left(\frac{x}{2}\right)} = 0 \quad 0 \leq x \leq \pi \)

c) For \( \alpha \): \( \tan(3\alpha) = -1 \quad 0 \leq \alpha \leq \pi \)

39. In each case, determine if the expression is defined. If the expression is defined, simplify it as much as possible.

a) \( \arcsin\left(-\frac{1}{2}\right) \)  
   b) \( \cos^{-1}(2\pi) \)  
   c) \( \tan\left(\tan^{-1}\left(\frac{3}{4}\right)\right) \)

d) \( \arccos\left(\cos\left(\frac{5\pi}{4}\right)\right) \)  
   e) \( \tan\left(\arcsin\left(\frac{20}{29}\right)\right) \)

40. Let \( f(x) = \begin{cases} 
2 + \cos\left(\frac{\pi x}{4}\right) & x \leq 2 \\
\frac{12 - 5x}{x + 2} & x > 2 
\end{cases} \)
a) Find $f(0)$.

b) Find $x$ such that $f(x) = 0$.

c) Which of the following statements is true? (select all that apply)

   ____ $f(x)$ is continuous at $x = 2$.
   ____ $f(x)$ has a jump (break) at $x = 2$.
   ____ $f(x)$ has a hole at $x = 2$.
   ____ $f(x)$ has a vertical asymptote at $x = 2$.

41. Find the value of $C$ that makes $q(t)$ continuous on $(-\infty, \infty)$.

\[
q(t) = \begin{cases} 
  C t^2 + 2 & t \leq -2 \\
  -2t + 8 & t > -2
\end{cases}
\]

42. Determine if each function is continuous on the given interval.

a) $g(\theta) = \frac{e^{\sin \theta}}{\cos \theta}$ on $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ 

b) $f(x) = 3x - x^{-1}$ on $[-1, 2]$

43. Find the exact values of the following limits:

a) $\lim_{z \to 4} \frac{28z - 7z^2}{z^2 - 2z - 8}$ 

b) $\lim_{t \to \infty} \frac{3t^2 + 5t^3}{7t^3 + t - 1}$

44. Use the graph of $f(x)$ at the right to find the following:

a) $f(5)$

b) $f(3)$

c) $\lim_{x \to 5} f(x)$

d) $\lim_{x \to 3} f(x)$
45. Suppose \( f \) and \( g \) are functions such that \( \lim_{{x \to 1}} f(x) = 8 \) and \( \lim_{{x \to 1}} g(x) = 2 \). Find the following:

a) \( \lim_{{x \to 1}} (2f(x) + 9) \)  

b) \( \lim_{{x \to 1}} \frac{g(x)}{(f(x))^2} \)  
c) \( \lim_{{x \to 1}} \cos(g(x)) \)

46. Find the following limits for \( f(x) = \frac{1}{1 + e^{\frac{1}{x}}} \).

a) \( \lim_{{x \to \infty}} f(x) \)  
b) \( \lim_{{x \to 1}} f(x) \)  
c) \( \lim_{{x \to 0^+}} f(x) \)  
d) \( \lim_{{x \to 0^-}} f(x) \)  
e) \( \lim_{{x \to 0}} f(x) \)

47. Factor completely (simplify the factors):

a) \(-30z^3(1 - 6z)^4 + 3z^2(1 - 6z)^5\)  
b) \(9 + 6e^x + e^{2x}\)

48. Simplify as much as possible:

a) \( \frac{b^n5^{n+1}}{5^nb^{n+1}} \)  
b) \( \frac{4-m^2}{3m-6} \) for \( m \neq 2 \)  
c) \( \ln(5e^y) \)

d) \( 10^{\log x} \)  
e) \( \frac{4(y+2)^{1/2} - 2y(y+2)^{-1/2}}{y+2} \)

49. Use algebra to solve for the indicated variable:

a) For \( u \): \( 3^{4u+1} = e^{1-2u} \)  
b) For \( p \): \( \sqrt{p^2 + 4} = 9 \)

c) For \( x \): \( \frac{5x^2 - 9x - 2}{x - 7} = 0 \)  
d) For \( t \): \( 7te^{5t} - 3t^2 e^{5t} = 0 \)

e) For \( h_2 \): \( A = \frac{1}{2}b(h_1 + h_2) \)  
f) For \( w \): \( \log(w - 3) + \log(w) = \log(28) \)

g) For \( x \): \( y = \frac{4 + 3x}{5 + 2x} \)  
h) For \( z \): \( 5z^3 - z^4 \geq 0 \)