**Business Mathematics II**  
Final Exam Study Guide

**NOTE:** This final exam study guide contains a small sample of questions that pertain to mathematical and business related concepts covered in Math 115B. It is not meant to be the only final exam preparation resource. Students should consult their notes, homework assignments, quizzes, tests, and any other ancillary material so that they are well prepared for the final exam.

**Questions 1-4** refer to the following data.

Data representing the numbers of injury automobile accidents in the town during the past few years have been plotted on the graphs below. A logarithmic trend line and an exponential trend line have been used to model the data.

1. Use the equation of the logarithmic trend line to predict the number of injury automobile accidents in the year 2002. The answer is:
   
   (A) Less than 7000  
   (B) Between 7000 and 8000  
   (C) Between 8000 and 9000  
   (D) Between 9000 and 10,000  
   (E) More than 10,000

2. Use the equation of the exponential trend line to predict the number of injury automobile accidents in the year 2040. The answer is:
   
   (A) Less than 100,000  
   (B) Between 100,000 and 200,000  
   (C) Between 200,000 and 300,000  
   (D) Between 300,000 and 400,000  
   (E) More than 400,000
3. In real world terms, explain why the prediction for the year 2040 given by the exponential trend line is or is not reasonable.

4. Using the $R^2$-value information provided in the graphs, which model would provide the better prediction for the number of injury automobile accidents in the years soon after 1999?
   (A) The logarithmic model because of the lower $R^2$-value
   (B) The exponential model because of the higher $R^2$-value
   (C) Since the $R^2$-value is not used for making predictions, nothing can be determined regarding which model is the better predictor
   (D) There is not enough information to draw a conclusion

5. Suppose the demand function for manufacturing a telephone is $D(q) = 200 - 0.2q$. If the fixed cost is $20,000 and it costs $50 to produce each telephone, determine the profit that could be made by selling 500 telephones.
   (A) $50,000
   (B) $45,000
   (C) $30,000
   (D) $5000
   (E) $100

6. If the demand function for a decorative vase is $D(q) = -0.0006q^2 - 0.002q + 450$, determine the price per unit that should be set in order to sell 700 vases.
Use the graph of the revenue and cost functions given below to answer questions 7 and 8.

7. Use the graph given above to estimate the number of units that should be produced in order to maximize profit. The number of units is approximately:

(A) 0  
(B) 200  
(C) 900  
(D) 1000  
(E) 1550

8. Use the graph given above to estimate the maximize profit. The maximum profit is approximately:

(A) $0  
(B) $45,000  
(C) $68,000  
(D) $98,000  
(E) $100,000
9. A company that produces dining room tables determines that their fixed costs are $100,000 and it will cost $180 to produce each table. How many tables could be produced for a total cost of $275,500? The total number of tables is:

(A) Less than 900  
(B) Between 900 and 950  
(C) Between 950 and 1000  
(D) Between 1000 and 1050  
(E) More than 1050

Suppose the demand function for a certain product is given by \( D(q) = -0.0005q^2 + 80 \). Use this function to answer questions 10 and 11.

10. Determine the largest possible quantity that could be produced using the demand function given above.

(A) 80  
(B) 400  
(C) 3578  
(D) 17,889  
(E) 160,000

11. Determine what should be inserted into the excerpt of Integrating.xlsm shown below in order to plot \( D(q) = -0.0005q^2 + 80 \) and estimate the total possible revenue.
Use the graphs of profit and marginal profit to answer questions 12 and 13. Assume no more than 1400 units are produced and sold.

12. On approximately what interval is $R(q) > C(q)$?
   
   (A) $[0, 1000)$
   (B) $(100, 1000)$
   (C) $[0, 550)$
   (D) $(100, 550)$
   (E) $[550, 1000)$

13. On approximately what interval is $MR(q) > MC(q)$?
   
   (A) $[0, 1000)$
   (B) $(100, 1000)$
   (C) $[0, 550)$
   (D) $(100, 550)$
   (E) $[550, 1000)$
A company estimates that the demand function for its product is given by $D(q) = -0.0002q^2 + 100$. Determine a formula for consumer surplus when 300 units are produced and sold.

\[ \int_{0}^{300} -0.0002q^2 + 100 \, dq - 82 \]

\[ \int_{0}^{300} q \cdot (-0.0002q^2 + 100) \, dq - 82 \]

\[ \int_{0}^{300} q \cdot (-0.0002q^2 + 100) \, dq \]

\[ \int_{0}^{300} -0.0002q^2 + 100 \, dq - 24,600 \]

\[ \int_{0}^{300} -0.0002q^2 + 100 \, dq - 24,600 \]

A company decides to sell helium balloons. Use the fact that the revenue function is $R(q) = -0.01q^2 + 150q$ and the cost function is $C(q) = 11,000 + 5q$ to answer questions 15 and 16.

15. Use the revenue and cost functions given above to determine formulas for the marginal revenue and marginal cost functions using the shortcuts for derivatives.

16. Use the formulas from question 15 to determine the number of balloons that would need to be manufactured and sold to maximize profit. The number of balloons is:

(A) Less than 7300
(B) Between 7300 and 7500
(C) Between 7500 and 7700
(D) Between 7700 and 7900
(E) More than 7900

17. Suppose the marginal revenue and marginal cost function for a product are $MR(q) = -0.075q + 150$ and $MC(q) = 45$, respectively. Determine whether revenue is increasing or decreasing at $q = 1500$ and whether profit is increasing or decreasing at $q = 1500$. At a quantity of 1500 units:

(A) Revenue and profit are both decreasing
(B) Revenue is decreasing and profit is increasing
(C) Revenue is increasing and profit is decreasing
(D) Revenue and profit are both increasing
(E) Cannot be determined

18. Suppose the marginal revenue and marginal cost function for a product are $MR(q) = -0.075q + 150$ and $MC(q) = 45$, respectively. Determine the quantity that maximizes profit.
19. The graphs of marginal revenue and marginal cost are show below.

Use the graphs to determine whether revenue, cost, and profit are increasing, decreasing, or constant at a quantity of 100 units.

(A) Revenue: Decreasing  
   Cost: Constant  
   Profit: Decreasing

(B) Revenue: Increasing  
   Cost: Increasing  
   Profit: Decreasing

(C) Revenue: Increasing  
   Cost: Constant  
   Profit: Decreasing

(D) Revenue: Increasing  
   Cost: Increasing  
   Profit: Increasing

(E) Revenue: Decreasing  
   Cost: Decreasing  
   Profit: Increasing

20. The demand function for a product is \( D(q) = -2q^2 + 60 \). Use a difference quotient with \( h = 0.001 \) to estimate the marginal demand when 5 units are produced.

(A) $119.96 per unit  
(B) $1 per unit  
(C) $0.04 per unit

(D) $20 per unit  
(E) $40 per unit
21. A company that produces mirrors for telescopes estimates the values for the following functions when 1200 mirrors are produced: \( R(1200) = 30,000 \), \( C(1200) = 23,000 \), \( MR(1200) = 400 \), and \( MC(1200) = 100 \). Due to a change in the economy, the revenue function decreased by 5000 and cost increased by 10\%. Determine the revenue, cost, marginal revenue, and marginal cost under the new economic conditions if 1200 mirrors are produced.

22. The cost for producing a new type of sunglasses is given by \( C(q) = 40,000 + 70q \). An investment of $9000 for new equipment would decrease marginal costs by 15\%. Determine a formula for the new cost function and new marginal cost function.

(A) \( C(q) = 49,000 + 70q \)
\[ MC(q) = 70 \]

(B) \( C(q) = 49,000 + 10.5q \)
\[ MC(q) = 10.5 \]

(C) \( C(q) = 49,000 + 70q \)
\[ MC(q) = 59.5 \]

(D) \( C(q) = 49,000 + 70q \)
\[ MC(q) = 70 \]

(E) \( C(q) = 49,000 + 59.5q \)
\[ MC(q) = 59.5 \]

23. Let \( f(x) = \frac{5x}{x+1} \). Use a difference quotient with \( h = 0.0001 \) to approximate \( f'(4) \). The value of \( f'(4) \) is:

(A) Less than –1.5

(B) Between –1.5 and –0.5

(C) Between –0.5 and 0.5

(D) Between 0.5 and 1.5

(E) More than 1.5

24. Let \( g(x) = 0.75^x + 2 \). Use a difference quotient with \( h = 0.001 \) to approximate \( g'(5) \). Round your answer to 4 decimal places.
25. Use the result from question 24 to determine the equation of the tangent line to the graph of \( g(x) \) at \( x = 5 \). Round your answer to 4 decimal places.

26. If \( h'(x) = m \), where \( m \) is a non-zero constant, which of the following statements is true about the formula for \( h(x) \)?

(A) \( h(x) = 0 \)
(B) \( h(x) = m \), where \( m \) is a non-zero constant
(C) \( h(x) \) is a non-constant linear function
(D) \( h(x) \) is a quadratic function
(E) \( h(x) \) is an exponential function

27. Let \( f(x) \) and \( g(x) \) be differentiable functions at \( x = -2 \), and suppose that \( f'(-2) = -4 \) and \( g'(-2) = 5 \). Determine the value of \( R'(-2) \) if \( R(x) = 3 \cdot f(x) + 6 \cdot g(x) \).

(A) \( R'(-2) = 18 \)
(B) \( R'(-2) = 1 \)
(C) \( R'(-2) = -6 \)
(D) \( R'(-2) = -18 \)
(E) Cannot be determined

28. Let \( f(x) \) and \( g(x) \) be differentiable functions at \( x = -2 \), and suppose that \( f'(-2) = -4 \) and \( g'(-2) = 5 \). Determine the value of \( R'(-2) \) if \( R(x) = 7 \cdot f(x) + 3x - 8 \).

(A) \( R'(-2) = -1 \)
(B) \( R'(-2) = -10 \)
(C) \( R'(-2) = -25 \)
(D) \( R'(-2) = -34 \)
(E) \( R'(-2) = -42 \)
29. Graphs of \( y = k(x) \) and the tangent line to the graph of \( y = k(x) \) at \( x = 1 \) are given below.

Use the graphs to determine \( k'(1) \).

(A) 0  (B) \( \frac{1}{3} \)  (C) 1  (D) 3  (E) None of the above

30. Let \( D(q) \) represent the price (in dollars per watch) at which \( q \) watches can be sold. Give a practical interpretation of \( D(200) = 320 \).

(A) When 200 watches have been manufactured, the price per watch should be $320.
(B) The price for 200 watches is $320.
(C) For every 200 watches manufactured, the price increases by $320 per watch.
(D) When 200 watches have been manufactured, the price increases by $320 when one more watch is manufactured.
(E) When 320 watches have been manufactured, the price per watch should be $200.
31. Let $D(q)$ represent the price (in dollars per watch) at which $q$ watches can be sold. Give a practical interpretation of $D'(200) = -4.56$.

(A) When 200 watches have been manufactured, the price per watch should be $-4.56.
(B) For every 200 watches manufactured, the price decreases by $4.56 per watch.
(C) For every 200 watches manufactured, the price increases by $4.56 per watch.
(D) When 200 watches have been manufactured, the price decreases by $4.56 when one more watch is manufactured.
(E) When 200 watches have been manufactured, the price increases by $4.56 when one more watch is manufactured.

32. Fill in the boxes of the screen capture in such a way that Solver would find a value for $q$ so that $D(q)$ is equal to $6$. 

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$q$</td>
<td>$D(q)$</td>
<td>$R(q)$</td>
<td>$C(q)$</td>
<td>$P(q)$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>105</td>
<td>$7$</td>
<td>$735$</td>
<td>$583$</td>
<td>$152$</td>
<td></td>
</tr>
</tbody>
</table>

Solver Parameters:
- **Set Objective**: 
  - **To**: Value Of: $6$
- **By Changing Variable Cells**: $q$
- **Subject to the Constraints**: 
- **Select a Solving Method**: GRG Nonlinear
- **Solving Method**: Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.
33. Fill in the boxes of the screen capture in such a way that Solver would find a value for $q$ which gives a maximum value for $P(q)$, subject to the constraint that $D(q)$ is less than or equal to $6$. 

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$q$</td>
<td>$D(q)$</td>
<td>$R(q)$</td>
<td>$C(q)$</td>
<td>$P(q)$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>105</td>
<td>$7$</td>
<td>$735$</td>
<td>$583$</td>
<td>$152$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
34. Fill in the boxes of the screen capture in such a way that *Solver* would find a value for *q* which gives a maximum value for $P(q)$ by using a reference to the marginal profit.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><em>q</em></td>
<td><em>D(q)</em></td>
<td><em>R(q)</em></td>
<td><em>C(q)</em></td>
<td><em>P(q)</em></td>
<td><em>MR(q)</em></td>
<td><em>MC(q)</em></td>
<td><em>MP(q)</em></td>
</tr>
<tr>
<td>2</td>
<td>105</td>
<td>$7</td>
<td>$735</td>
<td>$583</td>
<td>$152</td>
<td>$13.20</td>
<td>$8.70</td>
<td>$4.50</td>
</tr>
</tbody>
</table>

![Solver Parameters](image)

**Set Objective:**

- **To:** Max
- **Value Of:**

**By Changing Variable Cells:**

**Subject to the Constraints:**

- Make Unconstrained Variables Non-Negative

**Select a Solving Method:** GRG Nonlinear

**Solving Method:**

Select the GOG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.
Consider the function \( f(x) = 2x^5 - 3x^3 + 15 \) on the interval \([-2, 0]\). Use this function to answer questions 35-38, which relate to the steps for calculating the midpoint sum \( S_4(f, [-2, 0]) \).

35. Use the interval given above to determine the \( x \)-values \( x_0, x_1, x_2, x_3, \) and \( x_4 \) that divide the interval \([-2, 0]\) into four subintervals of equal length. What is the value of \( x_1 \)?

(A) \( x_1 = -2 \)
(B) \( x_1 = -1.5 \)
(C) \( x_1 = 0.5 \)
(D) \( x_1 = -1.6 \)
(E) \( x_1 = 0.4 \)

36. Use the information given above to determine the midpoints \( m_1, m_2, m_3, \) and \( m_4 \) of the four subintervals. What is the value of \( m_3 \)?

(A) \( m_3 = 0.5 \)
(B) \( m_3 = -0.25 \)
(C) \( m_3 = -0.5 \)
(D) \( m_3 = -0.75 \)
(E) \( m_3 = -1 \)

37. Use the information given above to determine the function value at each of the midpoints of the four subintervals. What is the value of \( f(m_4) \)? Round to 4 decimal places if necessary.

(A) \( f(m_4) = 10 \)
(B) \( f(m_4) = 14.1875 \)
(C) \( f(m_4) = 14.3125 \)
(D) \( f(m_4) = 14.8105 \)
(E) \( f(m_4) = 15 \)

38. Use the information given above to determine the midpoint sum \( S_4(f, [-2, 0]) \). Round to 4 decimal places if necessary.

(A) \(-28.75\)
(B) \(-14.375\)
(C) \(2.4219\)
(D) \(4.8438\)
(E) Cannot be determined
39. Let \( g(x) = 0.75^x + 2 \). Compute the midpoint sum \( S_4(g, [-12, 4]) \). The value of the midpoint sum is:

(A) Less than 40  
(B) Between 40 and 70  
(C) Between 70 and 100  
(D) Between 100 and 130  
(E) More than 130

NOTE: Questions 40-45 relate to material specific to Project 1 ideas. All conventions and units used in Project 1 are implied. These project questions are just a small sample of potential Project 1 questions and are not meant to be an inclusive list of all possible questions. Students should consult their notes, project quizzes, and their teacher for additional practice.

40. (Project 1) Suppose the potential national market for purchasing the UDMA CompactFlash cards for Project 1 is 130 million people. Data representing Test Markets 1-3 are provided below.

<table>
<thead>
<tr>
<th>Market Number</th>
<th>Market Size</th>
<th>Price</th>
<th>Projected Yearly Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,340,000</td>
<td>$165.00</td>
<td>2,893</td>
</tr>
<tr>
<td>2</td>
<td>2,530,000</td>
<td>$179.00</td>
<td>12,476</td>
</tr>
<tr>
<td>3</td>
<td>1,880,000</td>
<td>$194.00</td>
<td>10,678</td>
</tr>
</tbody>
</table>

Estimate the projected national sales for the price used in Test Market #1. The answer is:

(A) Less than 200,000 cards  
(B) Between 200,000 and 300,000 cards  
(C) Between 300,000 and 400,000 cards  
(D) Between 400,000 and 500,000 cards  
(E) More than 500,000 cards
41. (Project 1) Data representing the costs for producing the UDMA CompactFlash cards for Project 1 are provided in the table below.

<table>
<thead>
<tr>
<th>Fixed Cost For The Year (in millions)</th>
<th>$13.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable Costs</td>
<td></td>
</tr>
<tr>
<td>Quantity (in thousands)</td>
<td>Cost per unit</td>
</tr>
<tr>
<td>First</td>
<td>400</td>
</tr>
<tr>
<td>Next</td>
<td>700</td>
</tr>
<tr>
<td>Further:</td>
<td></td>
</tr>
</tbody>
</table>

Determine the total cost for producing 1,300,000 units.

(A) $239.6 million
(B) $226.0 million
(C) $130.709 million
(D) $117.109 million
(E) None of these

42. (Project 1) Which of the following functions can be negative?

(A) Demand
(B) Revenue
(C) Cost
(D) Marginal Revenue
(E) Marginal Cost

43. (Project 1) Suppose the demand function for producing the UDMA CompactFlash cards for Project 1 is \( D(q) = -0.00043q^2 + 0.026q + 510.3 \). Determine a formula for marginal revenue, \( MR(q) \), using the properties for derivatives.

(A) \( MR(q) = -0.00086q + 0.026 \)
(B) \( MR(q) = -0.00129q^2 + 0.052q + 510.3 \)
(C) \( MR(q) = -0.00129q^2 + 0.052q \)
(D) \( MR(q) = -0.00086q^2 + 0.026q + 510.3 \)
(E) \( MR(q) = -0.00086q^2 + 0.026q \)
44. \textbf{(Project 1)} Data representing the costs for producing the UDMA CompactFlash cards for \textit{Project 1} are provided in the table below.

<table>
<thead>
<tr>
<th>Fixed Cost For The Year (in millions)</th>
<th>$13.60</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Variable Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity (in thousands)</td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td>First 400</td>
</tr>
<tr>
<td>Next 700</td>
</tr>
<tr>
<td>Further</td>
</tr>
</tbody>
</table>

What is the marginal cost when 900,000 units have been manufactured?

(A) $13.60 per unit  
(B) $240 per unit  
(C) $160 per unit  
(D) $90 per unit  
(E) Cannot be determined
Test market sales data from seven test markets for the UDMA CompactFlash cards for Project 1 were collected. The data values were used to approximate the national sales values. The national sales values were plotted along with the quadratic demand trend line. The results are displayed in the graph below.

\[ D(q) = 0.000023q^2 - 0.321q + 422.62 \]

Use the equation of the demand function to determine the quantity (in thousands) that can be sold if the price is $279.99.

(A) 93.670 thousand cards
(B) 330.940 thousand cards
(C) 334.546 thousand cards
(D) 431.019 thousand cards
(E) 459.456 thousand cards
46. The p.m.f. values for a finite random variable $T$ are listed in the table below.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$-5$</th>
<th>$-2$</th>
<th>$3$</th>
<th>$12$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_T(T)$</td>
<td>0.32</td>
<td>0.24</td>
<td>0.14</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Determine the mean of $T$, $\mu_T$.

(A) 6.1  
(B) 2  
(C) 1.94  
(D) 1.525  
(E) 0.485

Use the following information to answer questions 47-49. Let $W$ be a binomial random variable with parameters $n = 20$ and $p = 0.10$. A screen capture of Excel’s `BINOMDIST` function is given below.

47. Which of the following formulas would compute $P(W \leq 4)$?

(A) `BINOMDIST(4, 20, 0.10, TRUE)`  
(B) $1 - \text{BINOMDIST}(4, 20, 0.10, \text{TRUE})$  
(C) `BINOMDIST(4, 20, 0.10, \text{FALSE})`  
(D) $1 - \text{BINOMDIST}(4, 20, 0.10, \text{FALSE})$  
(E) `BINOMDIST(5, 20, 0.10, \text{TRUE})`
48. Which of the following formulas would compute \( P(W = 4) \)?

(A) \( \text{BINOMDIST}(4, 20, 0.10, \text{TRUE}) \)
(B) \( 1 - \text{BINOMDIST}(4, 20, 0.10, \text{TRUE}) \)
(C) \( \text{BINOMDIST}(4, 20, 0.10, \text{FALSE}) \)
(D) \( 1 - \text{BINOMDIST}(4, 20, 0.10, \text{FALSE}) \)
(E) \( \text{BINOMDIST}(5, 20, 0.10, \text{TRUE}) \)

49. Determine the mean of \( W, \mu_w \). Round to 4 decimal places if necessary.

(A) 10
(B) 2
(C) 1.8
(D) 1.4142
(E) 1.3416
Use the following information to answer questions 50-53.

The p.d.f. and c.d.f. of a continuous random variable, $X$, are given by the following formulas.

$$f_X(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\frac{x}{2} & \text{if } 0 \leq x \leq 2 \\
0 & \text{if } x > 2
\end{cases}$$

$$F_X(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\frac{x^2}{4} & \text{if } 0 \leq x \leq 2 \\
1 & \text{if } x > 2
\end{cases}$$

The graphs of the two functions are given below.
50. Determine the formula for an integral that could be used to calculate \( P(0.8 \leq X \leq 1.6) \).

(A) \( \int_{0.8}^{1.6} \frac{x}{2} \, dx \)

(B) \( \int_{0.8}^{1.6} \frac{x^2}{4} \, dx \)

(C) \( \int_{0.8}^{1.6} \frac{x \cdot x}{2} \, dx \)

(D) \( \int_{0}^{2} \frac{x \cdot x}{2} \, dx \)

(E) \( \int_{0}^{2} \frac{x}{2} \, dx \)

51. Calculate \( P(0.4 \leq X < 1.3) \).

(A) 0.45  (B) 0.4  (C) 0.3825  (D) 0.32  (E) None of these

52. Use the graph of the p.d.f. to approximate the value of the mean. The mean is:

(A) Less than 1

(B) Equal to 1

(C) Between 1 and 1.5

(D) Equal to 1.5

(E) Between 1.5 and 2

53. Determine the formula for an integral that could be used to calculate the mean of \( X, \mu_X \).

(A) \( \int_{0}^{2} \frac{x \cdot x^2}{4} \, dx \)

(B) \( \int_{0}^{2} \frac{x^2}{4} \, dx \)

(C) \( \int_{0}^{2} \frac{2^2}{4} - \frac{0^2}{4} \, dx \)

(D) \( \int_{0}^{2} \frac{x}{2} \, dx \)

(E) \( \int_{0}^{2} \frac{x \cdot x}{2} \, dx \)
54. Which of the following integrals would verify that the function \( f(t) \) given by

\[
f(t) = \begin{cases} 
90t(1-t)^8 & \text{if } 0 \leq t \leq 1 \\
0 & \text{elsewhere}
\end{cases}
\]

is a valid p.d.f.?

(A) \( \int_{-\infty}^{\infty} 90t(1-t)^8 \, dt \)
(B) \( \int_{-\infty}^{\infty} t \cdot 90t(1-t)^8 \, dt \)
(C) \( \int_{0}^{1} t \cdot 90t(1-t)^8 \, dt \)
(D) \( \int_{0}^{1} 90t(1-t)^8 \, dt \)
(E) \( \int_{0}^{1} \left(t - \frac{2}{3}\right)^2 \cdot 90t(1-t)^8 \, dt \)

55. Suppose \( K \) is an exponential random variable with parameter \( \alpha = 6 \). What is the value of \( P(K = 6) \)? Round the answer to 4 decimal places if necessary.

(A) 1  (B) 0.6321  (C) 0.1667  (D) 0.0613  (E) 0

56. Let \( H \) be a uniform random variable on the interval \([0, 20]\). Which of the following calculations would correctly compute the probability that \( H \) is more than 13?

(A) \( F_H(13) \)
(B) \( 1 - F_H(13) \)
(C) \( \int_{13}^{20} F_H(h) \, dh \)
(D) \( \int_{14}^{20} f_H(h) \, dh \)
(E) \( 1 - F_H(14) \)

57. Let \( R \) be an exponential random variable with parameter \( \alpha = 4 \). What is the value of \( P(R \leq 8) \)? Round the answer to 4 decimal places if necessary.

(A) 0  (B) 0.0338  (C) 0.1353  (D) 0.5  (E) 0.8647
For questions 58-67, identify each integral as a probability, mean, or variance and determine its value. If necessary, round your answer to 4 decimal places.

58. Compute \( \int_{4}^{7} \frac{1}{8} \, dx \).

59. Compute \( \int_{6}^{11} \frac{1}{5} \, dx \).

60. Compute \( \int_{0}^{6} x \cdot \frac{1}{6} \, dx \).

61. Compute \( \int_{0}^{18} (x - 9)^2 \cdot \frac{1}{18} \, dx \).

62. Compute \( \int_{4}^{7} e^{-\frac{x}{8}} \, dx \).

63. Compute \( \int_{0}^{\infty} x \cdot \frac{1}{8} e^{-x/8} \, dx \).

64. Compute \( \int_{0}^{\infty} (x - 8)^2 \cdot \frac{1}{8} e^{-x/8} \, dx \).

65. Compute \( \int_{-\infty}^{\infty} \frac{1}{5\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x-12}{5} \right)^2} \, dx \).

66. Compute \( \int_{12}^{\infty} \frac{1}{5\sqrt{2\pi}} e^{-0.5 \left( \frac{x-12}{5} \right)^2} \, dx \).

67. Compute \( \int_{-\infty}^{\infty} x \cdot \frac{1}{5\sqrt{2\pi}} e^{-0.5 \left( \frac{x-12}{5} \right)^2} \, dx \).

68. Compute \( \int_{-\infty}^{\infty} (x - 12)^2 \cdot \frac{1}{5\sqrt{2\pi}} e^{-0.5 \left( \frac{x-12}{5} \right)^2} \, dx \).
Use the information provided below to answer *questions 69 and 71*.

Let $C$ be a finite random variable that gives the length, in seconds, of commercials sold by a local radio station. The *p.m.f.* of $C$ is given in the table below.

<table>
<thead>
<tr>
<th>$c$</th>
<th>15</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c(c)$</td>
<td>0.125</td>
<td>0.795</td>
<td>0.055</td>
<td>0.010</td>
<td>0.015</td>
</tr>
</tbody>
</table>

69. Determine the mean of $C$, $\mu_C$. The mean is:

(A) Less than 10  
(B) Between 10 and 30  
(C) Between 30 and 50  
(D) Between 50 and 70  
(E) Between 70 and 90

70. Determine the standard deviation of $C$, $\sigma_C$. Round your answer to 4 decimal places if necessary.

(A) 232.1494  
(B) 34.7890  
(C) 28.1745  
(D) 15.2364  
(E) 5.6325

71. Determine $P(Y \leq 75)$.

(A) 1  
(B) 0.975  
(C) 0.0325  
(D) 0.025  
(E) 0

72. Suppose $D$ is a binomial random variable with parameters $n = 86$ and $p = 0.62$. Determine the standard deviation of $D$, $\sigma_D$. Round your answer to 4 decimal places if necessary.

(A) 4.5013  
(B) 7.3021  
(C) 20.2616  
(D) 53.32
73. Let $X$ be a continuous random variable whose p.d.f. is given by

$$f_X(x) = \begin{cases} 
0 & \text{if } x < -1 \\
\frac{x^2}{3} & \text{if } -1 \leq x \leq 2 \\
0 & \text{if } x > 2 
\end{cases}$$

If $E(X) = 1.25$, which of the following integrals would correctly compute the variance of $X$, $V(X)$?

(A) $\int_{-1}^{2} (x - 1.25)^2 \cdot \frac{x^2}{3} \, dx$
(B) $\int_{-1}^{2} \frac{x^2}{3} \, dx$
(C) $\int_{-1}^{2} x \cdot \frac{x^2}{3} \, dx$
(D) $\int_{-\infty}^{\infty} (x - 1.25)^2 \cdot \frac{x^2}{3} \, dx$
(E) $\int_{-\infty}^{\infty} x^2 \cdot \frac{x^2}{3} \, dx$

74. A company collects a sample that contains the number of years its employees have been working at the company. Five sample values are shown below.

2, 7, 16, 9, 11

Determine the sample mean and sample standard deviation. Round your answer to 4 decimal places if necessary.

75. Let $C$ be the random variable that gives the number of customers who visit your business in a given day. If $\bar{C}$ is the random variable that is the mean of a random sample of size 16 days, compute the mean of $\bar{C}$, $\mu_{\bar{C}}$, if $\mu_C = 30$ and $\sigma_C = 6$.

(A) $\mu_{\bar{C}} = 0.375$
(B) $\mu_{\bar{C}} = 1.5$
(C) $\mu_{\bar{C}} = 1.875$
(D) $\mu_{\bar{C}} = 7.5$
(E) $\mu_{\bar{C}} = 30$
76. Let $C$ be the random variable that gives the number of customers who visit your business in a given day. If $\bar{C}$ is the random variable that is the mean of a random sample of size 16 days, compute the standard deviation of $\bar{C}$, $\sigma_{\bar{C}}$, if $\mu_C = 30$ and $\sigma_C = 6$.

(A) $\sigma_{\bar{C}} = 0.375$
(B) $\sigma_{\bar{C}} = 1.5$
(C) $\sigma_{\bar{C}} = 2.4495$
(D) $\sigma_{\bar{C}} = 1.875$
(E) $\sigma_{\bar{C}} = 7.5$

77. Suppose that $R$ is a random variable with a mean of 38.44 and a variance of 81, and let $S$ be the standardization of $R$. Find a value $b$ so that $P(S < 2) = P(R < b)$.

(A) $b = 200.44$
(B) $b = 93.4$
(C) $b = 85.88$
(D) $b = 56.44$
(E) $b = 21.4$

78. Let $B$ be a binomial random variable with parameters $n = 4$ and $p = 0.3$. The values of the p.m.f. of $B$ are given in the table below.

<table>
<thead>
<tr>
<th>$b$</th>
<th>$f_B(b)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2401</td>
</tr>
<tr>
<td>1</td>
<td>0.4116</td>
</tr>
<tr>
<td>2</td>
<td>0.2646</td>
</tr>
<tr>
<td>3</td>
<td>0.0756</td>
</tr>
<tr>
<td>4</td>
<td>0.0081</td>
</tr>
</tbody>
</table>

Determine the p.m.f. value for the standardization of $B$ (rounded to 4 decimal places) if $\mu_B = 1.2$ and $\sigma_B \approx 0.9165$. If $S$ is the standardization of $B$, what is the value of $P(S = 1.9640)$?

(A) 0.2401  (B) 0.4116  (C) 0.2646
(D) 0.0756  (E) 0.0081
79. Scores on the Graduate Management Admissions Test (GMAT) are normally distributed with a mean of 528 and a standard deviation of 112. What equation involving the \textit{NORMDIST} function from \textit{Excel} would need to be typed to calculate the probability that the score on the GMAT is less than 500? A screen capture of the \textit{NORMDIST} function is given below.

\begin{itemize}
  \item[(A)] \( = \text{NORMDIST}(500,528,112,\text{TRUE}) \)
  \item[(B)] \( = \text{NORMDIST}(500,528,112,\text{FALSE}) \)
  \item[(C)] \( = 1 - \text{NORMDIST}(500,528,112,\text{TRUE}) \)
  \item[(D)] \( = 1 - \text{NORMDIST}(500,528,112,\text{FALSE}) \)
  \item[(E)] None of these
\end{itemize}

80. Let \( M \) be the normal random variable that gives the starting salary for a graduate from the school of business. Assume that \( \mu_M = 38,142 \) and \( \sigma_M = 6595 \). If the 85\% confidence interval for the standard normal random variable \( Z \) is \(-1.44 < Z < 1.44\), determine an 85\% confidence interval for \( M \).
81. Suppose \( X \) is a continuous random variable with a mean of 74.6 and a standard deviation of 4.9. Let \( \mu_\bar{x} \) be the continuous random variable that is the mean of a sample of size 8. If the 85% confidence interval for the standard normal random variable \( Z \) is \(-1.44 < Z < 1.44\), determine an 85% confidence interval for \( \mu_\bar{x} \). Express the values to 3 decimal places.

(A) 73.769 < \( \mu_\bar{x} \) < 75.431
(B) 73.718 < \( \mu_\bar{x} \) < 75.482
(C) 72.249 < \( \mu_\bar{x} \) < 76.951
(D) 72.105 < \( \mu_\bar{x} \) < 77.095
(E) 67.544 < \( \mu_\bar{x} \) < 81.656

82. Two graphs of the p.d.f. for the standard normal random variable is given below along with two shaded regions. The first shaded region calculates \( P(-2 \leq Z \leq -1) \) and has an approximate area of 0.1359. The second shaded region calculates \( P(0 \leq Z \leq 1) \) and has an approximate area of 0.3413.

Use these graphs and the associated probabilities to determine the approximate value of \( \int_{-1}^{2} f_Z(z) \, dz \).

(A) 0.9544
(B) 0.8185
(C) 0.6131
(D) 0.4772
(E) Cannot be determined
83. The p.d.f. of a normal random variable, $X$, with a mean equal to 5 and standard deviation equal to 2 is plotted below.

Which of the following statements is **FALSE**?

(A) $\int_{-\infty}^{\infty} f_X(x) \, dx = 1$
(B) $\int_{-\infty}^{5} f_X(x) \, dx = 0.5$
(C) $\int_{1}^{6} f_X(x) \, dx = F_X(6) - F_X(1)$
(D) $F_X(5) = 1 - F_X(5)$
(E) $P(X < 3) < P(X \leq 3)$

84. Which one of the following statements is **FALSE** about the graph of a general normal random variable?

(A) The graph of the p.d.f. is symmetrical around the mean.
(B) The area under the graph of the p.d.f. on an interval containing $\pm 5$ standard deviations from the mean is approximately 1.
(C) The maximum height of the graph is approximately $\frac{0.4}{\text{standard deviation}}$.
(D) The graph crosses the x-axis.
(E) The area under the graph of the p.d.f. on an interval containing $\pm 1$ standard deviation from the mean is approximately 0.6827.
85. Which of the following 5 graphs correctly displays the graph of a general normal random variable whose mean is 4 and standard deviation is 2?

(A) 

(B) 

(C) 

(D) 

(E)
Let $X$ be a normal random variable with $\mu_X = 24$ and $\sigma_X = 3.2$. Which of the following screen captures correctly displays the information that would be needed to have the Excel function *Random Number Generation* create random values of $X$ in cells A1:F10?

(A) [Screen Capture A]

(B) [Screen Capture B]

(C) [Screen Capture C]

(D) [Screen Capture D]
Let $X$ be a normal random variable with $\mu_X = 35$ and $\sigma_X = 7.4$. Which of the following screen captures correctly displays the information that would be needed to have the Excel function $NORMINV$ create random values of $X$?

(A) 

<table>
<thead>
<tr>
<th>Probability</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>35</td>
</tr>
<tr>
<td>Standard_dev</td>
<td>7.4</td>
</tr>
</tbody>
</table>

(B) 

<table>
<thead>
<tr>
<th>Probability</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>35</td>
</tr>
<tr>
<td>Standard_dev</td>
<td>7.4</td>
</tr>
</tbody>
</table>

(C) 

<table>
<thead>
<tr>
<th>Probability</th>
<th>RANDBETWEEN(0, 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>35</td>
</tr>
<tr>
<td>Standard_dev</td>
<td>7.4</td>
</tr>
</tbody>
</table>

(D) 

<table>
<thead>
<tr>
<th>Probability</th>
<th>RAND()</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>35</td>
</tr>
<tr>
<td>Standard_dev</td>
<td>7.4</td>
</tr>
</tbody>
</table>
NOTE: Questions 88-93 relate to material specific to Project 2 ideas. All conventions and units used in Project 2 are implied. These project questions are just a small sample of potential Project 2 questions and are not meant to be an inclusive list of all possible questions. Students should consult their notes, project quizzes, and their teacher for additional practice.

(Project 2) Fifteen oil companies all bid on oil leases. The following data represent a small excerpt of the records on past bids. All monetary amounts are in millions of dollars. Use this information to answer questions 88 and 89

<table>
<thead>
<tr>
<th>Leases</th>
<th>Historical Signals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proven Value</td>
<td>Company 1 Signals</td>
</tr>
<tr>
<td>$ 273.6</td>
<td>$ 268.4</td>
</tr>
<tr>
<td>$ 153.7</td>
<td>$ 161.2</td>
</tr>
<tr>
<td>$ 189.4</td>
<td>$ 179.3</td>
</tr>
</tbody>
</table>

88.  (Project 2) Compute the errors for the 6 given signals.

89.  (Project 2) Compute the mean of the errors for the 6 given signals.

   (A) –$2.60 million and $1.30 million
   (B) $2.60 million and –$1.30 million
   (C) –$0.65 million
   (D) $0.65 million
   (E) $0 million

90.  (Project 2) Which of the assumptions in Project 2 allows you to assume that the mean of the errors is zero?

   (A) The same companies will all bid in the auction, and they will be the only bidders for the tracts.
   (B) The geologists employed by each of the bidding companies are all equally expert and, on average, they can estimate the correct values of leases.
   (C) Except for their means, the distributions of the signal values are all identical.
   (D) All of the companies act in their own best interests, have the same profit margins, and have the same needs for business. Thus, the fair value of a lease is the same for all companies.
Data representing the errors from four prior auctions involving four companies are provided in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Error 1</th>
<th>Error 2</th>
<th>Error 3</th>
<th>Error 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auction 1</td>
<td>$11.60</td>
<td>-$4.60</td>
<td>$7.30</td>
<td>$2.10</td>
</tr>
<tr>
<td>Auction 2</td>
<td>$13.70</td>
<td>$2.50</td>
<td>-$12.60</td>
<td>-$4.30</td>
</tr>
<tr>
<td>Auction 3</td>
<td>$2.60</td>
<td>$9.80</td>
<td>-$3.60</td>
<td>$4.40</td>
</tr>
<tr>
<td>Auction 4</td>
<td>-$7.90</td>
<td>$3.60</td>
<td>$6.10</td>
<td>-$15.20</td>
</tr>
</tbody>
</table>

Use the data in the table to answer questions 91 and 92.

91.  *(Project 2)* Determine the value of the winner’s curse using the table given above. (Round to 2 decimal places if necessary.)

(A) $0.97 million  
(B) $8.80 million  
(C) $10.30 million  
(D) $11.60 million  
(E) $13.70 million

92.  *(Project 2)* Determine the value of the winner’s blessing using the table given above. (Round to 2 decimal places if necessary.)

(A) $11.60 million  
(B) $5.85 million  
(C) $4.45 million  
(D) $2.95 million  
(E) $0.97 million
93. **(Project 2)** Assume that a simulation of 5000 sample auctions with 19 companies and a standard deviation of $17.35 million produced a Winner’s Curse of $29.52 million. If the standard deviation increased, determine how this change would impact the value of the Winner’s Curse. You should state whether the Winner’s Curse would increase, decrease, or stay the same and also provide a correct explanation regarding why the Winner’s Curse would increase, decrease, or stay the same.

(A) The Winner’s Curse would increase since the maximum errors in the 5000 auctions would increase due to a larger standard deviation.

(B) The Winner’s Curse would increase since the difference between the maximum errors and second largest errors in the 5000 auctions would increase due to a larger standard deviation.

(C) The Winner’s Curse would decrease since a larger standard deviation would lower the probability of winning the auction, so companies would want to subtract more to increase their profit.

(D) The Winner’s Curse would stay the same since all the errors in the 5000 auctions would be closer to zero due to a larger standard deviation.

(E) The Winner’s Curse would stay the same since the standard deviation has no effect on the Winner’s Curse.