Evaluate the following integrals exactly using the substitution method. In each problem you need to find (i) \( u \), (ii) \( du \), (iii) integral in terms of \( u \), (iv) limits of integration in terms of \( u \) (for the definite integrals), and then (v) evaluate the integral. Use proper notation.

1. \[ \int_0^\pi \cos^2\left(\frac{\theta}{5}\right)\sin\left(\frac{\theta}{5}\right)\,d\theta \]
   \( u = \) 
   \( du = \) 
   \( ___ \leq u \leq ___ \)

2. \[ \int (t+1)e^{5t+5}\,dt \]
   \( u = \) 
   \( du = \)

3. \[ \int_0^1 \frac{1+e^{3x}}{e^{3x}+3x}\,dx \]
   \( u = \) 
   \( du = \) 
   \( ___ \leq u \leq ___ \)

4. \( \int \frac{\sin(\ln ax)}{x}\,dx \), where \( a > 0 \)
   \( u = \) 
   \( du = \)
5. If \( \int \frac{dx}{1 + \cos x} = \tan \left( \frac{x}{2} \right) + c \), find \( \int \frac{1}{3 + 3 \cos \left( \frac{x}{4} \right)} \) exactly where \( c \) is a constant.

\[ w = \]
\[ dw = \]

6. If \( \int_{0}^{3} \frac{1}{1 + y^2} \, dy = k \), find \( \int_{0}^{1} \frac{1}{1 + 9x^2} \, dx \) in terms of \( k \).

\[ u = \]
\[ du = \]

\[ \_\_\_ \leq u \leq \_\_\_ \]

7. If \( \int_{0}^{\pi} x \, dx = \frac{\pi^2}{12} \), find \( \int_{0}^{\infty} \frac{x}{e^{5x} + 1} \, dx \) exactly.

\[ u = \]
\[ du = \]

\[ \_\_\_ \leq u < \_\_\_ \]