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Cultural and Linguistic Resources to Promote Problem Solving and Mathematical Discourse among Latino/a Kindergarten Students

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1 The second and third authors contributed equally to this paper.
A central tenet of the *Principles and Standards for School Mathematics* is that all students, even young children, should participate in solving problems and communicating their mathematical thinking (NCTM, 2000). Problem solving creates opportunities for children to construct understanding of important mathematical ideas (Hiebert et al., 1996), to engage in mathematical practices such as presenting explanations and comparing solutions (Cobb et al., 1993; Yackel & Cobb, 1996), and to develop positive dispositions towards the subject matter (Carpenter, Fennema, Peterson et al., 1989; Cobb, Wood, Yackel et al., 1991; Franke & Carey, 1997).

A substantial body of research has investigated the problem solving capacity of young children. As early as first grade (Carpenter, Fennema, Peterson et al., 1988, 1989; Carpenter, Hiebert & Moser, 1981; Fennema, Carpenter, Franke et al., 1996) and kindergarten (Carpenter et al., 1993; Outred & Sardelich, 2005; Warfield, 2001), children can solve a range of word problems, often by modeling the quantities and relationships involved. While prior research has documented a) that young children can solve problems, and b) that certain teacher practices seem to support students’ learning, with notable exceptions (i.e., Carey, Fennema, Carpenter & Franke, 1995; Villaseñor & Kepner, 1993), the majority of the research has been conducted in predominantly white, English speaking, middle class schools (e.g., Carpenter et al., 1993; Fennema, Carpenter, Franke et al., 1996; Warfield, 2001). Much less is known about how young Latino/a students, nearly half (45%) of whom are English language learners (Kohler & Lazarín, 2007), learn to solve and discuss mathematical problems, and equally important, about the knowledge, strategies, and practices that their teachers draw upon to support their understanding.

Given that Latino students are the fastest growing group in our nation’s public schools (Kohler & Lazarín, 2007), and the persistent gap in achievement between Latino/a students and
their white and Asian counterparts (NAEP, 2005), this is a critical void in the literature. A recent national assessment of 22,000 young children, specifically kindergarteners, reported that black and Latino/a students entered kindergarten with more limited mathematical knowledge and skills than their white peers, and made fewer gains in achievement over the course of the year (NCES, 2000). In particular, while low-income black and Latino/a students made some progress in basic mathematical skills (i.e., counting), at the end of the year they lagged even further behind white and Asian students on measures of more advanced mathematical knowledge such as solving simple addition and subtraction word problems (NCES, 2000). In other words, over the course of one year of formal schooling in kindergarten, the gap had widened (see also Jordan, Kaplan, Oláh & Locuniak, 2006, for similar findings).

While a widening “achievement gap” is certainly cause for concern, documenting the gap does little to inform our understanding of what actually happens in these kindergarten classrooms, or of the kinds of opportunities to learn (Tate, 1995, 2005) to solve and discuss mathematical problems that are available, or not available, to young Latino/a students (e.g., Oakes, 1990). We contend, as many others have argued, that the underachievement of lower income, minority students may be largely attributed to differential school and classroom-based experiences (e.g., Fryer & Levitt, 2004; Khisty, 1995; Oakes, 1990).

That being said, there is an urgent need to investigate the participation and learning of young Latino/a students in classrooms that do focus on problem solving and mathematical discourse. Specifically, research should document teaching practices that support low income and minority students’ success in problem-solving oriented classrooms (Boaler, 2002), including practices that address the specific learning needs of Latino/a students, many of whom are in the process of learning English.
In this article, we report on what happens when Latino/a kindergarten students in three classrooms have repeated opportunities to participate in solving and discussing basic word problems. We focus particular attention on the classroom practices and cultural and linguistic resources that teachers draw up to support students’ learning. We begin by presenting a set of theoretical tools that inform our work and analysis, and then briefly review prior work related to young children’s problem solving.

THEORETICAL TOOLS

Socio-cultural Perspective on Learning

Consistent with situated and socio-cultural perspectives on learning (Moschkovich, 2002; Nasir & Hand, 2006), we see learning mathematics as learning to participate in a community of practice (Lave & Wenger, 1991; Wenger, 1998) where students solve and discuss problems and reason about mathematical ideas. We see the process of learning to take on problem solving roles (e.g., problem solver, explainer of thinking, question poser) and engage in mathematical discourse practices as inseparable from what it means to know and do mathematics. Learning mathematics is inherently a social and cultural endeavor. As Nasir and Hand (2006) argue, “understanding learning requires a focus on how individuals participate in particular activities, and how they draw on artifacts, tools, and social others to solve local problems” (p. 450).

Language, Discourse and Learning Mathematics. As a primary tool for knowing and interacting with the world, language plays a key role in learning. It mediates thought and new understandings via socially situated interactions (Vygotsky, 1978). In mathematics classrooms structured around solving and discussing problems, attention to language is particularly important (Moschkovich, 2002). Children not only construct mathematical ideas through language-rich interactions (Yackel & Cobb, 1996), they learn to “speak mathematically” (Pimm,
which includes, but is not limited to, communicating their thinking, explaining solution strategies, and appropriating specialized mathematical vocabulary. Teachers play a critical role in providing students “access” to mathematical words and ways of talking (Khisty & Chval, 2002), by modeling mathematical discourse (Hufferd-Ackles et al., 2004) and providing students with repeated opportunities to use the language in context.

Given that language mediates learning, it is important to emphasize the role of students’ native language in supporting their mathematical understanding (Khisty, 1995, 1999). If young children are to solve problems and communicate their thinking about those problems it is reasonable to assume that they must first have opportunities to make sense of the problem, and that such sense making is facilitated by opportunities to work in their native language (or a language in which they are proficient), or to draw on their native language for support.

_Participation, Identity and Learning Mathematics._ Sociocultural perspectives on learning also draw attention to the relationships between participation and identity, specifically, how the ways that students participate can impact how they view themselves as learners, and vice versa (Nasir & Hand, 2006). When students have repeated opportunities to participate in ways that honor their experiences, ideas and particular ways of knowing, the learning can be transformative (Wenger, 1998). In mathematics classrooms, when teachers position students as knowledgeable and competent problem solvers who have something important to contribute to the classroom community, they not only support students’ understanding, but place them on a trajectory towards greater competence and participation (e.g., Empson, 2003). In such communities of practice, it is possible for students to create productive relationships with the discipline of mathematics (Boaler, 2002b; Cobb & Hodge, 2002) and identities that imagine greater competence in the future (Wenger, 1998).
Cultural Knowledge, Practices, and Learning Mathematics. Socio-cultural perspectives view culture as a dynamic and socially constructed way of knowing and interacting with the world. Culture is not a set of traits, customs, and celebrations, but the practices that people engage in, what “people actually do and what they say about what they do” (González, Andrade, Civil, & Moll, 2001, p. 118). Research has argued that incorporating the cultural practices, knowledge and skills of students’ households and communities into classroom activity can enhance students’ learning (Civil, 2006; González, et al., 2001; González, Moll, & Amanti, 2005; Moll, 1992) by allowing students to tap into familiar, everyday experiences and practices - ways of talking, solving problems, understanding, and making sense of the world. Specific to mathematics, teachers and researchers have worked collaboratively to uncover mathematical funds of knowledge in the households of low-income and minority students, and to design specific classroom activities or units of study that integrate these knowledge bases (for examples see Civil, 2002, 2006; Civil & Kahn, 2002; Kahn & Civil, 2001).

In summary, socio-cultural perspectives emphasize the importance of language, social interaction, cultural knowledge and experiences, and participation in communities of practice as key elements of understanding and supporting children’s learning. From a socio-cultural perspective, understanding how young Latino/a students learn to solve problems and explain their thinking requires attending to these aspects of instruction. However, existing research on young children’s problem solving rarely gives these aspects of instruction the attention we feel is necessary to address the needs of Latino students. Below we comment on this research.

PRIOR RESEARCH ON YOUNG CHILDREN’S PROBLEM SOLVING

An extensive body of research has documented that young children can solve a broad range of simple word problems by directly modeling the actions and relationships involved
Problem Solving among Latino/a Kindergartners

Carpenter et al. (1993) found that kindergartners who had repeated opportunities to solve a variety of basic word problems demonstrated remarkable success on an end-of-the-year-assessment. Almost half of the 70 students interviewed used valid strategies on all of the problems, which included multiplication, division, and multi-step problems, and the majority of students were successful on the most basic problem types (e.g., subtraction). It is important to note that all teachers participated in a professional development program, *Cognitively Guided Instruction*, focused on the development of children’s thinking about basic operations. Although teachers were not provided with specific guidelines for instruction, they were encouraged to use information about children’s thinking to plan and adapt problem-solving tasks. Teachers generally did not show students how to solve problems, but instead provided students with concrete materials, such as counters, that they could use to model the problem.

It is relevant to note, given the focus of our research, that students in the Carpenter et al. (1993) study were predominantly white (72% in one school and 77% in another) and from middle, or upper-middle class communities. That being said, research with *first* grade students in more ethnically diverse urban schools (57% to 99% students of color) has documented similar findings. When students have repeated opportunities to solve and discuss word problems they perform significantly better on assessments of problem solving and number facts than peers in more traditional classrooms (Villaseñor & Kepner, 1993; see also Carey, Fennema, Carpenter & Franke (1995) for a study in schools with predominantly African American student populations).

In summary, extensive research has described how teachers draw upon knowledge of children’s mathematical thinking as they organize problem solving based instruction, as well as the impact of their instruction on young children’s learning (Carey et al., 1995; Carpenter et al.,
It is not our intent to reiterate the value of such instruction here. Rather, what we still know little about is how young Latino/a students, including those who are ELLs, learn to solve problems and communicate their mathematical thinking. Specifically, while we know that teachers draw upon knowledge of children’s mathematical thinking to support student learning, how teachers draw upon other knowledge bases, such students’ language, culture, or home experiences, as resources to support their understanding warrants further investigation. Our study helps to address this critical void in the literature. In the sections that follow, we describe the setting and participants of our study, as well as our methods for generating and analyzing data.

METHODS

Participants

We focused on three kindergarten classrooms in schools with predominantly Latino/a student populations (87%, 75%, and 72%, respectively), where almost all students (over 90%) qualified for free or reduced lunch. In Ms. Arenas’ classroom, all students were native Spanish speakers with varying degrees of English language proficiency. Ms. Arenas, also a native Spanish speaker, followed a dual-language model of instruction (Lindholm-Leary, 2001); almost all of her mathematics lessons were in Spanish. Ms. Field, who was trained in ESL (English as a Second Language) strategies, taught mathematics in English. Close to half of Ms. Field’s students were English language learners and the rest were native English speakers. Ms. Perales, who was trained in bilingual education, taught mathematics in both Spanish and English, using English mainly to support students who were learning Spanish as a second language. Approximately half of Ms. Perales’ students were native Spanish speakers and half spoke

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All names are pseudonyms.
English as their first language. All three teachers taught in all-day kindergarten programs, and had approximately 18 to 20 students in their class.

We selected these three classrooms because teachers had participated in professional development focused on young children’s mathematical thinking (i.e., Cognitively Guided Instruction, see Carpenter, Fennema, Franke et al., 1999), and they were interested in conducting problem-solving lessons with their students. For two of the teachers, Ms. Arenas and Ms. Field, this was their first year using what they had learned about children’s thinking to plan and implement problem solving tasks. The other teacher, Ms. Perales, had previous experience.

Data Collection and Analysis

Pre and Post Assessments. As one method of documenting students’ learning, we conducted pre and post task based clinical interview assessments (Ginsburg et al., 1983). At the beginning of the year (October), we selected 21 students (7 from each classroom) who represented a range of achievement levels to participate in a pre assessment, which included both counting and problem-solving tasks (e.g., join, separate, multiplication, and division word problems). Table 1 displays selected pre-assessment items. All problems were presented orally, and students had access to multiple tools (counters, cubes, paper and pencil). Interviewers re-read each problem as many times as needed, and clarified specific information if the child asked. After each problem, children were asked to explain their solution strategy.

---Insert Table 1 Here---

At the end of the year (May), we administered a post-assessment that included a broader range of problem types, modeled after the problems included in the Carpenter et al. (1993) study. (Table 2 displays selected post assessment items). A total of 45 students participated in the post-assessment, approximately 15 from each classroom. All students who received parental consent
participated in these interviews. As in the pre-assessment, students had access to multiple tools and after each problem were asked to explain their reasoning. On both assessments, students were interviewed in their dominant language, either Spanish or English. All interviews were conducted by trained members of the research team, and video-taped for later analysis.

We coded students’ responses to pre and post assessment items using the coding scheme reported in Carpenter et al. (1993). Each response was coded in terms of a) the strategy that the child used (i.e., direct modeling, counting, recalled fact), b) whether the strategy was valid or invalid, and c) whether the answer was correct. Valid strategies were those that absent a miscount would have resulted in a correct answer. We used descriptive statistics, ANOVA measures and general linear models (GLM) to interpret the findings from these data.

Classroom observations. The second source of data involved on-going observations of problem solving lessons in the three classrooms. We visited Ms. Arenas’ and Ms. Field’s classrooms on a bi-weekly basis to observe and/or videotape instruction. Because of distance and resources, Ms. Perales’ classroom was observed once a month. For each teacher, a typical lesson lasted between 30 and 45 minutes. Over the course of the year, we videotaped and fully transcribed at least six lessons in each classroom. Transcription attended not only to what was said, but to other aspects of interactions, such as gestures and use of tools and representations.

Teacher interviews. Each teacher was interviewed once in the fall and once in the spring. The interviews asked teachers to reflect on the strategies they used to support students’ learning, the knowledge that they drew upon to design and implement tasks, and their thoughts about the understanding of particular students. All interviews were audio-taped and transcribed.
Analysis of observation and interview data. Classroom and interview transcripts were coded according to the principles of grounded theory (Strauss & Corbin, 1990). This involved chunking the data into meaningful units, and then coding selected statements or interactions using words or phrases that specifically addressed the research questions (Erlandson et al., 1993). For example, classroom observations were coded with particular attention to specific teacher and student actions related to solving and discussing mathematical problems. We used a computer-based qualitative research tool, TAMS® Analyzer, to code all data. To establish reliability (Miles and Huberman, 1994; Patton, 2002), transcripts were coded by at least two members of the research team. Differences in interpretation were discussed until agreement was reached. The TAMS® Analyzer tool allowed us to search across transcripts to establish recurring patterns, or themes, related to particular instructional practices. Each theme was then triangulated across data from all participants, and across data of various forms (i.e., classroom observations, field notes, teacher interviews) (Erlandson et al., 1993).

In the sections that follow, we begin with a brief overview of the pre-assessment results, and then describe a typical problem-solving lesson in each of the kindergarten classrooms. We continue with a detailed analysis of specific instructional practices that teachers used to support students in learning how to solve problems and explain their reasoning. We conclude with a discussion of students’ performance on the post assessment measures.

FINDINGS

Analysis of Pre Assessment Measures

Typical of many kindergarten classrooms, children began the year with a range of mathematical experiences and proficiencies. On a Kindergarten Developmental Progress Record [KDPR] that teachers administered during the first month of school, most students (80%)}
Problem Solving among Latino/a Kindergartners

were able to count a small set of objects (up to 8 items) and to recognize some numerals from 1 to 10. However, less than half of the students could count past 10 or 15, and there were several students in each classroom who at the beginning of the year did not count past 2 or 3, or demonstrate one-to-one correspondence. As Ms. Field noted, “When I look over those [KDPR] screenings, and I remember testing some of these kiddos, and they didn’t know more than 1, 2, 3. … Another part of it was how far could they count, and there were quite a few kids that couldn’t even count to ten which with my first time [teaching] in kindergarten really amazed me.”

In terms of problem solving almost half of the 21 students (9 out of 21, or 43%) who participated in the pre-assessment successfully solved a basic addition problem, and slightly more than half (11 of 21 students, or 52%) solved a basic subtraction problem (table 1, problems a and b). Given that many students began the year with somewhat limited counting skills, and that national assessments have found that only 4% of kindergartners are able to solve basic addition and subtraction problems at the beginning of the year (NCES, 2000), we find students’ success on these easier problem types to be remarkable, and further evidence of the typically underestimated problem solving capacity of young children. However, other items on the pre-assessment, such as multiplication (14% solved), partitive and measurement division (28% and 23% solved, respectively) and compare problems (0% solved) were significantly more difficult.

Strategy use

Of all problems solved correctly, the vast majority (82%) was solved using direct modeling strategies (i.e. students modeled the quantities and relationships involved using fingers, objects, or drawings). Only 4 of the 21 students used more advanced counting strategies, and these strategies were all used on easier problem types (i.e., Join Result Unknown). It is important to note that students’ performance on the pre-assessment was relatively consistent
across the three classrooms. On average, students solved 2 of the 7 items correctly, ranging from an average of 1.67 items correct in Ms. Field’s class to 2.4 items correct in Ms. Perales’s class.

In summary, students’ performance on pre-assessment measures reflected a range of understandings that is typical of kindergartner students. If anything, some students began the year with less developed number concepts and skills than kindergarten teachers might expect.

Portrait of Instruction

Ms. Arenas, Ms. Field and Ms. Perales drew on a variety of instructional formats in their problem solving lessons. Common to all lessons was that the teacher orally presented a word problem, and then encouraged students to solve the problem in ways that made sense to them, which is consistent with the principles of Cognitively Guided Instruction (Carpenter et al. 1999). Students often used concrete materials, such as counters and cubes, or drew pictures on small white boards to support their reasoning. After most students had solved a problem, teachers facilitated a group discussion in which multiple students shared their strategies.

In some instances, teachers worked on problem solving with one small group of students at a time. Typically, teachers grouped students heterogeneously, although there were instances when specific students were grouped together because the teacher felt they would benefit from working on a certain type of problem. On other occasions, teachers presented the entire class with a problem, and students worked individually or with a partner to generate a solution. In addition to problem solving, each of the teachers used center activities that involved building and counting sets to support students’ number sense and counting skills. It is important to note that teachers did not wait until students had mastered a set of “basic skills” to begin problem solving lessons. They introduced problem solving at the beginning of the year, and used contextual problems to strengthen students’ number-related concepts and skills. As Ms. Arenas noted:
given this general portrait of instruction, we now shift our focus to a detailed analysis of the specific practices that teachers used to help students solve problems and communicate their thinking. while documenting every practice is beyond the scope of this paper, we focus here on the cultural and linguistic resources that teachers drew upon to support students’ learning. we conclude our findings section with an overview of students’ performance on the post-assessment.

analysis of classroom practices

1. generating mathematical problems through authentic, storytelling conversations

One practice common to the teachers in our study was the use of authentic, “story-like” conversations to generate mathematical problems. teachers often presented stories in an informal, conversational manner, including rich contextual information, and inviting students to respond with questions or comments. by framing problem solving around telling and investigating stories, teachers drew upon ways of talking and negotiating meaning that were familiar to children. that is, all children have experience listening to stories and using stories to communicate meaning, which is particularly prevalent in some latino families (delgado-gaitan, 1987; villenas & moreno, 2001). the dialogic nature of the stories that teachers told invited students to enter the situation and imagine themselves as active participants and contributors.

For example, ms. arenas typically began her lessons by telling students to listen carefully (“Fíjense, amorcitos”), because she was about to share a story (“Les voy a contar otra historia”). In one conversation, she built a story around a current classroom activity: decorating and filling Easter baskets. the structure of the problem she generated (9÷3) reflected an actual classroom event in which the student teachers needed to equally distribute nine eggs among three people.
Ms. A: *Vamos a ver otra. Fíjense, que estamos ya comprando los huevitos de pascua para sus canastas. Los huevitos de pascua. Y tenemos –* (Let’s do another one. Listen, you know we’re already buying the little Easter eggs for your baskets. The Easter eggs. And we have --- )

Julieta: *Maestra, mi mamá y una prima ya compraron las canastas. (Teacher, my mom and a cousin already bought the baskets.)*

Ms. A: *¿Ya compraron las canastas?* (They already bought the baskets?)

Julieta: *¡Sí!* (Yes!)

Ms. A.: *Ah, Ms. Maribel y Ms. Aracely (two classroom assistants) ya las van a empezar a decorar. ¿Verdad? Las canastas de pascua. Entonces, adentro les vamos a poner unos huevitos. (Oh, Ms. Maribel and Ms. Aracely (classroom assistants) are going to start decorating them now. Right? The Easter baskets. Then, inside, we are going to put some little eggs.)*

Alonso: *¿Maestra? ¿Maestra?* (Teacher? Teacher?)

Ms. A.: *¿Sí?* (Yes?)

Alonso: *Mi mamá me va a comprar una canasta de basquetbol. (My mom is going to buy me a basketball basket.)*

Ms. A.: *¿Para la pascua?* (For Easter?)

Alonso: *[nods]*

Ms. A.: *¡Ay, qué bonito!* (Oh, how nice!)

S: *Y maestra, mi mamá ya le echó dulces. (And teacher, my mom already put the candies in.)*

Ms. A.: *¿Ya le echó huevitos? Pues ahora, Ms. Anita y Ms. Crystal (student teachers) trajeron 9 huevitos. (She already put in little candy eggs? Well, today, Ms. Anita and Ms. Crystal (student teachers) brought 9 little [candy] eggs.)* …

Ms. A.: *Escuchen. Pero los quieren repartir, escuchen. Primero escuchen. 9 huevitos trajeron. Pero los quieren repartir entre Ms. Maribel, Ms. Aracely y Ms. Maggie. ¿Cuántos le van a quedar a cada una?* (Listen. But they want to share them, listen. First listen. They brought 9 eggs. But they want to divide them between Ms. Maribel, Ms. Aracely, and Ms. Maggie. How many is each one going to get?)

Interestingly, although this practice of generating mathematical problems through “storytelling” conversations was not explicitly discussed in the professional development that teachers received, it was typical of lessons that we observed across the three classrooms. For example, Ms. Field used a conversation about a popular recess activity - looking for interesting rocks on the playground – to generate a multiplication problem for students to solve. As the conversation unfolded, she created a story about three friends who each find three rocks and
place them in their pockets. Students’ task was to figure out how many rocks they had altogether.

In a similar example, Ms. Perales engaged students in a story about Mariela (a student in the class) and the shoes that she had at home in her closet. She discussed with students what a “pair” of shoes meant, taking off her own shoes to demonstrate, and posed questions such as, “If Mariela had 6 shoes, how many pairs would that be?” Mariela added to the story, saying she just had 4 shoes at home, because she was wearing one pair of shoes to school. Students then used Mariela’s contribution to figure out how many pairs of shoes she had at home.

Teachers commented that they viewed sharing and constructing stories as a way of drawing on students’ cultural knowledge and experiences. For example, Ms. Arenas noted, “Yes, I always try [to relate to their cultural experiences] and that is what I am doing with the CGI. I have been trying to give my stories. Other teachers put their own stories. A lot of stories are very cultural.” Using stories to connect students’ cultural knowledge and experiences with their mathematical activity has been documented in prior research (Lo Cicero, Fuson, & Allexsaht-Snider, 1999; Lo Cicero, De la Cruz, & Fuson, 1999). In what follows, we extend this work by contributing a more detailed analysis of how this practice supports young Latino/a kindergarten students as they learn to solve and discuss mathematical problems.

a. Stories that reflect familiar contexts invite students to draw upon lived experiences and funds of knowledge to make sense of mathematical ideas. Teachers rarely presented “generic” problems (e.g., Sara had 3 apples. Johnny gave her 5 more. How many does she have now?) or problems that were based on unfamiliar contexts. In fact, we only identified 5 such problems across all the lessons we observed. Instead, the stories reflected community events (e.g., going to the fair), family practices (e.g., purchasing fruit at a local market), shared classroom experiences (e.g., field trips, celebrations), or common play activities (e.g., games of marbles,
sharing toys with friends). Teachers seemed to be familiar with some of the practices that families engaged in, and how these practices might relate to mathematics. As Ms. Arenas noted:

[Children] bring rich experiences in going to the market with their parents. ... The market experience, the open market experience, [experience] with the money and how much do you pay for this or that. ... I know the kids know, maybe they can estimate how many strawberries are in that basket, because they have had the experience. ... They have other cultural experiences too. A lot of times they plant with their parents and even counting the seeds or transferring the seeds is math[ematics].

When teachers employed familiar ways of talking (i.e., stories), and used relevant contexts to frame mathematical ideas, this practice invited students to draw on their own experiences and funds of knowledge as they assumed an active role in making sense of and solving problems.

b. The familiar, narrative structure of stories scaffolds students’ explanations. Another way that teachers used stories to support students’ learning was by drawing on story structures to scaffold students as they learned to explain their thinking. When students struggled to explain their ideas, teachers often reminded them of the story context, and then used the narrative as a framework to guide students as they explained the steps they took to solve the problem. For example, Ms. Field presented the following story about a game of marbles between Sarita and Kyle: “Sarita had 8 marbles, and then Sarita gave some of those marbles to Kyle. She gave them away. And now she only has 4 left. So how many did she give to Kyle? Go ahead, try it.” She repeated the story several times, clarifying the quantities involved, and then allowed students time to work. After several minutes, Penny volunteered to share her solution with the class.

Ms. Field: Sure, come on up Penny. Everybody eyes up here. Tell us what you did.

Penny: First I started with 4 and then I started with 4 more then I counted then it made 9, and then I counted, and then ---- (she pauses, seems uncertain, and looks up at Ms. Field)

Ms. Field: Okay Penny, wait a minute, let me tell you the problem one more time. We said that Sarita had 8 marbles, and then she gave Kyle some, and she had 4 left. So how many did she give him?
Problem Solving among Latino/a Kindergartners

Penny: 4, 4. (Points to her picture. She has drawn a line of 8 marbles, with 4 marbles on one side of her white board and 4 marbles on the other).
Ms. Field: So she gave him 4 marbles?
Penny: Yes.
Ms. Field: And then how many did she have?
Penny: She gave [him] 4 and had 4 more left. Cause [it’s] like 4 and 4 is 8. (Points to the two groups of 4 on her board)

Ms. Field then restated Penny’s solution, emphasizing that Sarita started with 8 marbles, she gave away 4 and then had 4 left, and that Penny figured out that she gave away 4 because she knew that 4 and 4 is equal to 8. In this episode, we see how Ms. Field drew Penny back into the story context (“Wait a minute…. we said that Sarita had 8 marbles…”) when Penny seemed unsure about how to explain her strategy. Then, Ms. Field asked focused questions about various parts of the story (“So how many did she give him?”) as a way of guiding Penny to use the story as a resource to explain her thinking. In the end, when Ms. Field restated Penny’s ideas, she again offered Penny a model of how she might use the story to frame her explanation.

As we compared lessons across the year, we noticed that while teachers initially provided substantial guidance and modeling to help students communicate their reasoning, as the year progressed, students began to contribute clearer and more complete explanations. We suspect that repeated interactions in which teachers used the narrative structure of stories to scaffold students’ explanations (e.g., Ms. Field’s interaction with Penny) contributed to this growth. We do not mean to imply that explanations that are framed around events in a story are in some way more desirable than explanations that focus strictly on number relationships and operations. Rather, we argue that stories, which are familiar to children both in their form, and in case of the stories told by teachers in our study, in their content, provided a narrative structure that guided young children as they learned to explain their solution strategies.
c. **Stories help students learn to represent mathematical ideas and connect multiple representations.** Teachers also used stories to support students as they learned to represent mathematical ideas and to connect multiple representations (e.g., drawings, symbols, objects) of a given situation. The following episode from Ms. Perales’ classroom illustrates how the teacher continuously referred to the story to clarify the meaning of different representations. After students worked on the problem – which was about equally distributing 15 toy cars among three friends — Brenda volunteered to share her solution with the class.

**Brenda:** *I put 15 right here, 15 lines, and I put 1 line in each [circle], one for Beto, one for David, and one for Juan. (she gestures how she “passed” out one tally, representing a car, to each friend.) Cinco. Cinco! {Five. Five!} (meaning that each friend got a total of 5 cars)*

**Ms. Perales:** *Cinco. Juan, David y Beto, todos van a agarrar cinco, cada uno igual. Equally. Good job! {Five …. Juan, David and Beto, they’re all going to get 5, each one the same. Equaly. Good job!}*

**Ms. Perales:** *Okay, this is how Brenda did it. (walks up to the chalk board) Look. There are three friends. Let’s try this. David, Beto and Juan. (Writes the first letter of each boy’s name on the board.) Uno, dos, tres. (and draws a large circle under each letter.) Why do you think I put the circles under there?*

**Mariela:** *Because he gots [sic] carritos (little toy cars) for everyone.

**Ms. Perales:** *Okay, I put the circles to remind me, there are his, these are his, and these are his (points to each circle as she says, “his”). (Then draws a row of 15 lines (the cars) along the bottom of the board. This mimics the way that Brenda has represented the problem.)*

As the discussion continues, Brenda directs Ms. Perales to distribute one car (a tally) to each boy, and then to erase three tallies from the row of 15. Ms. Perales follows her instructions, drawing lines to link one tally to each boy’s circle, and then re-representing the tally inside the circle with a dot, just as Brenda had done. When Brenda comes to the board to continue enacting her solution, Ms. Perales stops to clarify the meaning of her representation for other students.

**Ms. Perales:** *What is she putting in here?*

**Yanitza:** *Cars!*

**Ms. Perales:** *Carritos. Cars. And then how come she is taking them away? How come she is putting some and then taking some away? (points to where she has erased 3 tallies from the row of 15).*
Mariela: Because three and three and three. For David, 3 for Beto and 3 for Juan. (Indicating that so far, Brenda has distributed 3 cars to each boy).
Ms. Perales: Okay, she is giving each kid a car.

When Brenda finished representing her solution, Ms. Perales asked students how many cars each boy received and they enthusiastically responded, “Cinco! Cinco! Y Cinco!”

Ms. Perales: ¡Sí! Hay cinco carritos aquí adentro (points to one boy’s circle, and writes the number ‘5’ below it), hay cinco carritos aquí adentro (points to the next circle, and again writes ‘5’), y hay cinco aquí (points to final circle, and writes ‘5’ below it). {Yes! There are 5 carritos here inside [the circle], there are 5 here inside, and there are 5 here}

Ya sabemos que podemos contar de cinco. ¿Quién puede contar de cinco?

Yantiza: 5, 10, 15!
Ms. Perales: ¿Hay 15? {Are there 15?}
Students: ¡Sí! {Yes!}

In this example, we see how Ms. Perales repeatedly referred to the story context to help students make sense of the various components of Brenda’s representation (e.g., “What is she putting in [the circle]? … Okay, she is giving each kid a car.”) She then carefully pointed back and forth between a group of 5 tallies and the number ‘5’ as she noted that each representation stood for the 5 cars that each boy received (e.g., “Hay cinco carritos aquí adentro.”) Teachers’ use of stories to help students make sense of different representations may have been especially helpful for ELL students, because each representation created an additional opportunity for students to make sense of the mathematical ideas.

In summary, teachers frequently used authentic, “story-like” conversations to generate mathematical problems. The use of relevant stories not only created opportunities for students to draw on cultural knowledge and experiences to make sense of problems, but it also scaffolded their explanations and supported their understanding of multiple mathematical representations. In the next section, we describe a second theme related to teachers’ instructional practices.
2. Scaffolding students’ ability to communicate their thinking through strategic use of positioning and questioning coupled with access to multiple resources.

Each of the teachers in our study spoke explicitly about the importance of mathematical discourse in their classrooms. They wanted students to communicate their thinking, and they were aware that learning to communicate mathematically could be challenging for young children, particularly children who were in the process of learning the language of instruction. Teachers described a variety of practices that they drew upon to address these challenges. For example, Ms. Field noted that creating a safe environment that encouraged students to take risks seemed to support mathematical communication in her classroom:

What’s good is that they’re [English language learners] experimenting and they might not know the exact term but they are trying to use it. And having a safe environment where nobody is allowed to laugh at anybody – I think that really helps the communication also. Even the ones that are learning the language, the risk takers like Rogelio, have made so many gains. He doesn’t care if he makes a mistake, he’s trying.

Ms. Arenas added that for some students, opportunities to hear peers explain strategies were important.

[I] support[ed] them first by guiding them because they didn't know what to do. … And then modeling from other students, because some of those students mastered that ability before and they were pretty good models of how to explain and verbalize their strategies. They became models to other students.

Teachers also supported students’ participation in mathematical discourse by encouraging the use of multiple resources to communicate ideas. For example, when teachers orally introduced a problem, they also used gestures, held up fingers to represent quantities, they pointed to relevant objects in the room, and in some cases translated particular words or phrases. In one sense, their actions were aimed at supporting students’ understanding of the problem. At the same time, their actions modeled how students might draw upon multiple resources to explain their own ideas. In fact, we noticed that almost invariably students did use a variety of resources to explain their
Problem Solving among Latino/a Kindergartners

thinking. For example, the use of concrete materials, fingers, drawings, and gestures to re-enact strategies during class discussions was quite common, and we contend, enhanced students’ ability to contribute their ideas and make sense of the ideas of others. Another way teachers supported students’ participation in mathematical discourse was by modeling mathematical ways of talking. Teachers frequently restated students’ strategies, and in doing so, used more mathematically precise language (e.g., “Ah, you know what you just did, you counted by sixes,” or “Oh, so you are saying that you counted on, you started at 4 and then counted on, 5, 6, 7, 8.”)

We include these brief examples to demonstrate that teachers drew on a variety of instructional practices to support Latino/a kindergarten students as they learned to communicate their thinking. Our intent is not to focus on these practices, which though productive, are not unique to the teachers in our study, and have been well documented in prior research (e.g., Author, in press; Echeverria, Vogt, & Short, 2004; Khisty & Chval, 2002; Moschkovich, 1999). Rather, we intend to highlight how teachers scaffolded students’ ability to communicate their thinking through the intentional use of positioning and questioning strategies. To demonstrate the power of these practices, we present in-depth analyses of episodes from two classrooms.

In the first example, students in Ms. Arenas’ class have solved a problem about Diego (one of their peers), and a toy plane that he wants to buy for 10 dollars. Since Diego only has 6 dollars, students need to determine how much more money he needs to purchase the plane.

Belén is the first student to share her strategy.

Ms. Arenas:  Vamos a empezar ya con Belén, a ver Belén, explícanos cuántos dólares más necesita Diego. {We are going to start now with Belén. Let’s see Belén, explain to us how many more dollars Diego needs.}

Belén:  Cuatro. {Four}

Belén: *Porque primero puse estos* (points to a row of 10 tallies she has drawn on her white board), *y luego puse estos* (pointing to a second row of 6 tallies that she has drawn directly beneath the row of 10), *y luego conté estos* (points to the “extra” four tallies in the row of 10). {First I put these, and then I put these, and then I counted these.}

Ms. Arenas: *Muy bien. ¿Cuántos dólares costaba el avión? Miren todos lo que hizo Belén pues.* {Very good. How many dollars did the plane cost? Everyone look at what Belén did.}

Belén: 10.

Ms. Arenas: *Uno, dos.* {One, two.} (Gestures that Belén should count the tallies that she drew.)

Belén: *Uno, dos, tres, cuatro, cinco, seis, siete, ocho, nueve, diez.* {One, two, three, four, five, six, seven, eight, nine, ten.}

Ms. Arenas: *El avión costaba 10.* {The plane cost 10 [dollars].}

Belén: (writes the number 10 on the large white board)

Ms. Arenas: *¿Y cuántos tenía Diego?* {And how many [dollars] did Diego have?}

Belén: *Seis dólares.* {Six dollars.}

Ms. Arenas: *Miren todos cuántos tenía Diego. Cuéntelo.* {Look [everyone] at how many Diego had. Count them.}

Belén: *Uno, dos, tres, cuatro, cinco, seis.* {One, two, three, four, five, six.}

Ms. Arenas: *Y miren cómo lo hizo Belén. Los fue poniendo abajo de lo que costaba el avión.* (Counts back and forth between row of 10 tallies and row of 6 tallies.) *Y luego, cómo supiste que le faltaban 4 dólares?* {And look at how Belén did it. She was putting them [the tallies that stood for the money Diego already had] below what the toy plane costs. And then, how did you know that he needed 4 more dollars?}

Brianna: *Porque yo los conté rápido, conté estos* (points to four “extra” tallies in the row of 10) *que faltaban.* {Because I counted them quickly and then I counted these that were left over.}

Ms. Arenas: *Muy bien, cuántos le faltaban a Diego, uno, dos, tres, cuatro.* (Points to each of the “extra” four tallies in the first row as she counts, 1, 2, 3, 4.) {Very good, how many does Diego still need – one, two, three, four.}

In this example, Ms. Arenas began by positioning Belén as a competent problem solver who had something important to explain to members of the classroom community, including herself (e.g., “Vamos a empezar con Belén, a ver Belén, expícanos... porque lo hiciste más rápido que Ms. Arenas.”) She then extended an open invitation for Belén to explain how she figured out that Diego needed four more dollars to buy the plane (e.g., “A ver Belén, ¿Cómo lo hiciste?”) We found that beginning discussions by first positioning students as competent, and then using open-
ended questions to invite students to share their ideas was typical of all three teachers’ practice. In fact, questions that either invited students to explain their thinking or pushed for more details about something that a student already said were the most frequently occurring teacher action codes across all of the lessons that we analyzed. However, when students seemed to struggle to articulate their ideas, or, when they explained their strategy in a way that may have been comprehensible to the teacher, but not to other students in the classroom (as was probably the case with Belén), teachers often responded by shifting to more focused, closed questions, and then returning to open-ended probes later in the interaction.

For example, Bélen described her strategy by stating that first she “put these” and then “these” and then she “counted these.” Understanding her explanation required attention to the physical representations that she referred to, and even then, her reliance on indexical language somewhat obscured the meaning she intended to communicate. We suspect that Ms. Arenas understood Belén’s explanation. Even so, for the benefit of other students, as well as Belén, she responded with a series of closed questions aimed at helping Belén to clearly articulate the meaning of her tally mark representation (e.g., “¿Cuántos dólares costaba el avión? … ¿Y cuántos tenía Diego?). She then modeled for Belén an explanation that highlighted a key aspect of her strategy: lining up tally marks to compare the total cost of the plane with the amount of money that Diego had (e.g., “Y miren como lo hizo Belén. Los fue poniedo abajo de lo que costaba el avión.”) It is important to note that even as Ms. Arenas shifted to more closed, fill-in-the-blank type questions, she continued to assign competence to Belén’s ideas by repeatedly inviting other students to attend to her explanation (e.g., “Muy bien. …miren todos lo que hizo Belén pues.) Ultimately, Ms. Arenas returned to an open-ended probe, again asking Bélen how

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3 We use *closed questions* to refer to fill-in-the-blank type questions that elicit a one-word response, such as a number or a statement of “yes” or “no.”
she knew that Diego needed four more dollars. At this point, Belén offered a clearer explanation of how she counted the ‘leftover’ tallies, and got four.

This pattern of shifting from open-ended questions to closed probes and then back to open invitations to explain ideas was apparent in all three classrooms. Since it was particularly evident in interactions where students initially struggled to articulate their reasoning, we see this strategic use of questioning, coupled with an explicit positioning of students’ ideas as valuable and important, as a notable way that teachers scaffolded students’ ability to communicate their mathematical thinking. It is critical to note that we see these two practices – strategic use of questioning and explicit positioning - as interrelated; they work together to support students’ participation in mathematical discourse. Merely shifting between open and closed questions, without positioning students as competent members of the classroom community (e.g., Empson, 2003), would not have been as effective, we contend. Furthermore, not only do these practices support one another, but their productiveness is related to other aspects of instruction described above, such as access to multiple resources to communicate ideas.

We continue now with a second example, from Ms. Field’s classroom. This example further demonstrates how strategic use of questioning and positioning supported young Latino/a students’ ability to communicate their thinking. In this lesson, students solved a simple addition problem about two girls in the class, Amalia and Natalie, who joined their collections of marbles (3+4=7). After several other students described their strategies, Ms. Field invited Amalia, an English language learner, to share her solution with the class.

Ms. Field: Would you tell us what you did, Amalia?
Amalia: I put a 3 and 4 (points to two sets of squares she has drawn on her white board)
Ms. Field: (Holds up Amalia’s board for other students to see). Everyone look at Amalia’s. Cap your marker. Amalia, tell us what you did.
Amalia; I did two, uh … square and square. 4 squares and 3 squares (Points again
to the two sets of squares she has drawn.)

Ms. Field: Okay, how did you get your answer? What did you do?
Amalia: (pause, no response)
Ms. Field: What’s your answer?
Amalia: (no response, she is looking at her white board)
Ms. Field: This is good. Did you count, or how did you do it?
Ms. Field: Show us.
Amalia: I count. 1, 2, 3.
Ms. Field: Good ---
Amalia: And I count 4. 1, 2, 3, 4.
Ms. Field: And then, what’s your answer?
Amalia: Uh … (pause)
Ms. Field: How did you -- (points to two sets of squares that Amalia has drawn on her white board) – did you count them?
Amalia: Yeah.
Ms. Field: Show me, how did you do it?
Amalia: I – the other one (points to a large hundreds chart on the chalkboard)
Ms. Field: Oh, you used this?
Amalia: Yeah.
Ms. Field: Come on up and show us how you did it. Everyone, let’s fold your hands and let’s look and see what Amalia is doing.
Amalia: (walks up to stand next to the hundreds chart)
Ms. Field: Show us.
Amalia: 1, 2, 3. (points to each of the numbers as she counts). And that be 4. (pause) Because I, uh (pause).
Ms. Field: So you had 3 (points to number 3 on the hundreds chart) and then you knew you had 4 more. So you counted 4 more. So count 4 more. 1 – (points to number 4 on the hundred chart)
Amalia: 1, 2 –
Ms. Field: 3 – (continues to point at the numbers as Amalia counts on)
Amalia: 3, 4.
Ms. Field: So what was your answer?
Amalia: 4 – is this one. (points to the number 7 on the hundreds chart)
Ms. Field: 7 (nods head). Very good, very good. Thank you.

As in the previous example, Ms. Field began by first positioning Amalia as someone who had an important contribution to make (e.g., “Everyone look at Amalia’s”), and then extending an open invitation for Amalia to share her strategy with the class (e.g., “Amalia, tell us what you did.”) Also like Ms. Arenas, Ms. Field shifted between open-ended and more focused, closed probes in response to Amalia’s apparent uncertainty about explaining her thinking. For instance,
when Amalia did not respond to her invitation to explain how she arrived at an answer, Ms. Field asked a more focused question, “Did you count?” to which Amalia responded affirmatively, “I – I count. I count.” Ms. Field then returned to a more open-ended question: “Show me, how did you do it?”

We find it significant that when teachers shifted to closed probes, as was the case in this example, they typically did not continue to use such probes throughout the remainder of the interaction. Instead, they used such questions strategically: to confirm their interpretation of a student’s action, to establish that a student understood the quantities involved in the problem, or to clarify parts of a student’s representation. Once a student had responded to such probes, and thereby clarified some aspect of his or her strategy, teachers returned to more open elicitations of students’ ideas. This pattern is significant, because it continually created opportunities for students to communicate their thinking in meaningful ways (i.e., beyond a one word response), even for students who struggled to articulate their ideas, like Amalia.

Also important in this example is that throughout the interaction, Ms. Field continued to position Amalia’s ideas as important, and to express confidence in her ability to communicate her reasoning to her peers. Given that Amalia was just beginning to learn English, these repeated expressions of confidence - evident in the repeated invitations that Ms. Field extended – are even more significant. It is relevant to note that other aspects of Ms. Field’s instruction, in particular, students’ access to a variety of resources to communicate her ideas, also scaffolded Amalia’s participation in this discussion. Specifically, the opportunity to use a hundreds chart to demonstrate for her peers how she started at 3, and then counted on 4 to get 7, was significant.

In summary, the kindergarten teachers in our study worked diligently to create a safe environment for discussion that honored multiple forms of communication and encouraged
students to use a variety of resources to articulate their ideas. Students knew that they would be encouraged and expected to explain their thinking as an integral part of the problem-solving process. More specifically, we found that teachers scaffolded students’ ability to communicate their thinking through the strategic use of questioning by positioning students as competent problem solvers who had important ideas to contribute. When coupled with other important aspects of instruction, such as access to multiple resources to communicate ideas, these practices expanded students’ opportunities to participate in mathematical discourse.

The previous sections described specific instructional practices that teachers used to support kindergarten students as they solved problems and discussed their thinking. To further document the potential impact of these instructional strategies on student learning, we conclude our findings with an overview of students’ performance on the post assessment.

Analysis of Post Assessment Measures

As previously noted, 45 students across the three classrooms participated in a problem solving based post assessment in May of kindergarten. Relative to the percentage of students who correctly solved particular problems on the pre-assessment, students demonstrated remarkable growth. A significant majority of students successfully solved the most basic join and separate problems (80% and 73%, respectively), and the join change unknown, multiplication, and partitive division problems were each solved by approximately half of the students (56%, 49% and 43%, respectively). The most difficult problems for students were those involving comparison and division with a remainder (problems g and i in Table 2), and even so, approximately one-fourth of the students solved each problem correctly, and an even greater percentage (33%) used a valid strategy. Interestingly, the multi-step problem, which one might
assume to be too difficult for young children, was correctly solved by 44% of the students. Table 3 presents a summary of students’ performance on various post assessment items.

----Insert Table 3 here----

It is worth noting that students’ performance on the join result unknown and separate result unknown problems was comparable to that of the bilingual, Latino/a first graders who solved similar problems in Secada’s (1991) study. Moreover, kindergartners were more successful on a comparable join change unknown problem (55% solved, as compared to 29% of first graders)\(^4\). In addition, the kindergartners in our study solved a much broader range of problems than what the results of a national assessment of 22,000 kindergartner students might predict (NCES, 2000). While the NCES study found that only 18% of kindergartners demonstrated the ability to solve simple addition and subtraction word problems by the end of the year, and that only 2% could solve basic multiplication and division problems, the students in our study solved similar problems at much higher rates. For example, 71% of students solved either a multiplication or a division problem, and 44% correctly solved both types of problems. While details about the instruction these kindergartners received were not provided in the NCES report, we can assume that most students were not in classrooms that emphasized solving and discussing problems. This once again demonstrates the value of providing young children with opportunities to solve problems in ways that make sense to them, and the need to document how teachers structure such opportunities to support all students’ learning.

**Strategy use.** Similar to the pre-assessment, and consistent with the findings of previous research (Carpenter et al., 1993), students used direct modeling strategies to solve the majority of

\(^4\) Students in Secada’s (1991) study did not solve multiplication, division, compare, or multi-step problems, and thus a comparison of those problem types is not possible. It is also important to note that the instruction students received was not necessarily focused on solving and discussing problems.
problems (84%). However, in contrast to the pre-assessment, approximately half of the students (49%) used an advanced strategy (e.g., either a counting strategy or a recalled number fact) to solve at least one problem. Thus not only did students make significant progress in their ability to make sense of and solve a variety of word problems, but they began to use more advanced problem solving strategies, particularly on the easier problem types.

*Students’ explanations.* Whereas students’ explanations at the beginning of the year were often vague or incomplete, many students produced clear, and more mathematical descriptions of their thinking on the post assessment. For example, Dalia solved a join change unknown problem (Ana has 7 dollars, how many more does she need to have 11?) by drawing 11 tally marks, crossing out 7, and then counting those that remained to get 4. She then explained her thinking:

Dalia: Primero Ana quiere comprar un avión que cuesta 11 dólares. Y le faltan...
(counting the tally marks she did not cross out) 1, 2, 3, 4.
Interviewer: Como supiste que le faltan 4 [dólares]?
Dalia: Puse 11, y quite los 7 (points to the 7 tally marks that she crossed out), y luego le faltan 4 (points to the 4 remaining tally marks).

*Dalia: First Ana wants to buy a plan that costs 11 dollars. And she needs (counting to the tally marks she did not cross out) 1, 2, 3, 4.
Interviewer: How did you know that she needs 4 more [dollars]?
Dalia: I put 11, and then I took away 7 (points to the 7 tally marks that she crossed out), and then she needs 4 [more] (points to the 4 remaining tally marks).*

Dalia’s explanation is typical of the kind of explanations that students provided on the post-assessment in that she restated key features of the story (e.g., “Ana quiere comprar un avión que cuesta 11 dólares”), reiterated the steps of her strategy (e.g., “Puse 11, y quite los 7”), and used language that described relationships between quantities (e.g., “Le faltan 4”)

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*Analysis of pre/post assessment gains.* While a problem by problem comparison of students’ performance on the pre and post assessments is complicated -- the post assessment

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5 While an in-depth analysis of students’ explanations is beyond the scope of this article, we do present an analysis of students’ use of language to communicate their thinking in another report, see Author (2007).
included a broader range of problem types, and problems that were similar in structure involved larger numbers – for the 21 students who participated in both assessments, we compared their success on seven “matched” items. In all cases but one, the matched items were identical in structure and similar in context, but the post assessment version included larger numbers (e.g., 3 pockets with 2 pennies in each (3x2) versus 3 bags with 6 marbles in each (3x6)). While students solved an average of only 2 of the 7 items correctly on the pre-assessment, they solved an average of 4 of the matched items correctly on the post assessment. In other words, even though the items were more difficult, students were able to solve twice as many. It is also significant that these gains were apparent across students who began the year at different levels of mathematical proficiency. In fact, the six students who demonstrated “limited number concepts and skills” on initial kindergarten screenings achieved the greatest gains, solving an average of 3.2 of the 7 problems on the post-assessment, as compared to an average of less than 1 problem on the pre-assessment (a 45% gain). We find this result significant, as it demonstrates that students who begin the year with low levels of number-related skills can benefit substantially from reform-oriented mathematics instruction that is organized around solving and discussing problems.

Differences between classes. In our analysis of the post-test results, we noticed that on particular problem types, students in Ms. Arenas’ class performed substantially better than students in the other two classrooms. We used a general linear model to analyze variance in the overall means between Ms. Arenas’ students and students in the other classrooms. While the difference in overall means was significant (p < .001), meaning that in general, Ms. Arenas’ students performed significantly better on the post-assessment measure, further analysis of variance on each item revealed an insignificant difference on the most basic problem types (e.g., join result unknown and separate result unknown) and a significant difference on many of the
more challenging problems, such as multiplication, measurement and partitive division, and compare problems. Table 4 displays the results of this analysis.

Given these significant differences, stating the percentage of all students (n=45) that solved a particular problem does not provide an accurate reflection of the performance of students in Ms. Arenas’ class. For that reason, we include an additional table (Table 5) that displays post assessment results for Ms. Arenas’ students. This table also compares Ms. Arenas’ results to those of the kindergarten students in the Carpenter et al. (1993) study who solved a similar set of problems. While across all three classrooms students were somewhat less successful on the more difficult problems than students in the Carpenter study, Ms. Arenas’ students demonstrated a similar level of success on almost every problem type. For example, 63% of her students correctly solved the comparison problem, as compared to 67% of the students in the Carpenter study. In some instances, Ms. Arenas’ students actually did better, which we might attribute to the slight difference in the size of the numbers. For instance, 75% of Ms. Arenas’ students solved a partitive division problem that involved dividing a set of 15 into 3 equal groups (15÷3), while 70% of students in the Carpenter study solved a slightly more difficult, but still comparable problem that involved dividing 20 into 4 equal groups (20÷4).

---Insert Table 5 here---

Construct for Interpreting Differences: Opportunity to Learn

To support our analysis of the differences that we noticed between classrooms, we drew on Tate’s (1995, 2005) discussion of Opportunity-to-Learn (OTL). Tate (2005) proposed a three-part framework for highlighting key aspects of instruction that have been shown to impact students’ learning opportunities (p. 15). What follows is a summary of his model.

1. Content Exposure (e.g., How much time do students spend on a particular topic? Do they cover the topic in depth, or superficially?)
2. Content Emphasis (e.g., Which particular concepts and skills does the teacher decide to emphasize?)
3. Instructional Delivery (e.g., To what extent does the teacher use pedagogical strategies that effectively meet students’ learning needs?)

We found the framework particularly useful in analyzing the different opportunities to learn to solve problems that were created in each of the classrooms. Specifically, it drew our attention to the content of the problem solving opportunities (i.e., the kinds of problems students solved, and how often, the numbers used, and the time spent on each problem). We conjectured that Ms. Arenas’ students may have experienced greater success on some of the more challenging problem types because they had more opportunities to solve a broader range of problems.

To investigate this conjecture, we randomly selected a subset of our observations from each teacher’s classroom (three from the fall, and three from the spring), and for each lesson, noted the total number of problems that teachers presented to students, along with the specific problem types and numbers that were used. Table 6 displays the results of our analysis.

---Insert Table 6 Here---

Consistent with our conjecture, we found that Ms. Arenas’ lessons included a wider range of problem types and numbers, and a more equitable distribution between the most basic (JRU, SRU) and more challenging problem structures. For example, while only 42% of the problems that Ms. Arenas posed were Join Result Unknown or Separate Result Unknown, these basic problem types made up 67% and 68% of the problems in Ms. Perales and Ms. Field’s lessons, respectively. Interestingly, with the exception of measurement division, the problem types that evidenced a significant difference in performance across classrooms (see Table 4) were the same problems that Ms. Arenas presented more frequently than the other teachers. Moreover, in any given lesson, Ms. Arenas’ students had opportunities to solve approximately 5 problems, as compared to an average of 3.35 problems in the other classrooms. Additionally, while Ms.
Perales and Ms. Field reported that students solved word problems once, or sometimes twice a week, Ms. Arenas’ students participated in an average of 3 problem solving lessons per week. We conjecture that such enhanced opportunities-to-learn to solve problems contributed to the increased success that Ms. Arenas’ students demonstrated on the post assessment.

Another key difference between Ms. Arenas’ classroom and that of the other teachers was that all of Ms. Arenas’ students received mathematics instruction in their first language, Spanish. In each of other classrooms, approximately half of the students were solving problems in a language that they were in the process of learning. We conjecture that this may have impacted students’ opportunities to learn in several ways. First, when Ms. Arenas posed a problem, all students had access to a range of language-based tools such as explanations from the teacher and previous experiences that they could draw upon to make sense of the problem and underlying mathematical idea. Moreover, since language is inseparable from identity, the fact that Ms. Arenas’ instruction fully embraced each student’s home language may have helped students to see themselves as valued and competent participants in the classroom community, which in turn supported their learning (Khisty, 1999). This is not to say that the other teachers discouraged students from using their native language. Ms. Field frequently counted with students in Spanish, and encouraged students to speak Spanish with peers or her instructional assistant; Ms. Perales, whose lessons were mostly in Spanish, often dialogued with native English speakers about their strategies in English. Our intent is to highlight that only in Ms. Arenas’ classroom did instruction fully embrace and reflect each child’s native language (Cummins, 2001).

Finally, we noted a marked difference in the pacing of the lessons across classrooms. We suspect that since Ms. Arenas’ students all understood the language of instruction, she spent less time clarifying words, contexts, and ideas, which created more time for students to work on
solving problems. Given the uniqueness of the linguistic environment in each classroom, we can only conjecture about how differences in the language of instruction may account for differences in students’ achievement (e.g., Thomas & Collier, 2002). We suspect that it is one factor that may have contributed to the increased success of Ms. Arenas’ students.

Although Ms. Arenas’ students were more successful on some problem types, we want to emphasize that students in all three classrooms demonstrated remarkable growth in their ability to solve and discuss basic word problems, and achieved at higher levels than other studies might predict (NCES, 2000). We highlight the differences between the classrooms not to discount the work of Ms. Perales or Ms. Field, but to describe how the expanded opportunities-to-learn to solve and discuss problems that characterized Ms. Arenas’ instruction may have resulted in even greater achievement gains.

CONCLUSION

The achievement gap between Latino/a students and their white and Asian counterparts (NAEP, 2005) cannot be ignored, and it is important that the research community reports on the many social, political, economic and educational factors that have led to that gap. Equally important, however, is a focus on accomplishment. When low-income Latino students have repeated opportunities to solve and discuss mathematical problems, in classroom environments that draw upon cultural and linguistic resources to support their understanding, the gains are impressive, as this study documents. The aim of our research was not only to document students’ learning, but also to better understand the specific instructional practices and cultural and linguistic resources that teachers employed to support these young students.

Issues of class have reluctantly entered the discussion around equity in reform mathematics education. Specifically, some researchers have argued that children of low SES
backgrounds experience difficulty with the open-ended nature of problem solving, reasoning and communication (Lubienski, 2000). While not explicitly promoting a deficit orientation, the idea does assume that some students lack the richness of outside experiences and the sense of personal efficacy necessary to deal with the ambiguity inherent in problem solving situations. Boaler (2002a) counters these arguments and refocuses our attention on the critical role of teachers’ practices. She notes that students’ interactions with problem-solving based curricula are mediated by teacher moves that either support student understanding or leave students at a loss for what to do next. Because of the crucial mediating role of the teacher in promoting equity, Boaler argues that the field of mathematics education “is in need of additional examples of particular teaching practices that reduce inequalities” (p. 240). She notes that there is a lack of research on teaching practices with low SES students that help them develop the socio-mathematical norms necessary to be successful in mathematics classrooms that are structured around solving and discussing problems.

We agree with urgency of Boaler’s call, and present this study as one example of research that begins to document teaching strategies that support the successful participation of Latino/a students, including many who are English language learners, in reform oriented mathematics instruction. Not only do we contribute a detailed analysis of how particular instructional practices may have supported students’ learning, but we focus explicitly on practices that attend to the critical role of culture, language, and participation. While we can only conjecture about the impact of a specific practice, the learning gains that students demonstrated on post-assessment measures suggest that repeated opportunities to participate in classrooms organized around these practices were consequential.

Finally, we conclude with several suggestions for future research related to young
Latino/a students’ mathematics learning. Future studies might investigate the following questions, among others: 1) Differences in Latino/a students’ mathematics performance given different linguistic environments (i.e., classrooms in which mathematics is taught mainly in Spanish versus classrooms where mathematics is taught in English only); 2) The longitudinal impact of meaningful problem-solving experiences in kindergarten as Latino/a students progress through the elementary grades, and 3) The impact of repeated opportunities to solve and discuss problems on young children’s mathematical identities (their relationships to the discipline) and their emerging sense of mathematical agency.

Acknowledgements

We would like to thank other members of our research team, in particular Richard Kitchen, Edgar Romero, and Havens Levitt for their assistance in conducting pre and post assessment interviews and classroom observations. We offer a special thanks to the three teachers who so graciously allowed us into their busy classrooms with patience and humor, and all of the children who enriched our lives with their enthusiasm for learning. Most of all, we are indebted to our colleague, Alan Tennison, who spent countless hours in data collection and professional support in Ms. Field’s classroom. This study would not have been possible without his efforts.
Table 1. Selected Pre-Assessment Items (English Version)

<table>
<thead>
<tr>
<th>Problem Structure</th>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Join Result Unknown</td>
<td>a. Maya has 6 candies. Her brother gives her 3 more candies. How many candies does Maya have now?</td>
</tr>
<tr>
<td>Separate Result Unknown</td>
<td>b. Jason has 10 pennies. He loses 4 of them. How many pennies does Jason have now?</td>
</tr>
<tr>
<td>Multiplication</td>
<td>c. Javier has 3 pockets. He puts 2 pennies in each pocket. How many pennies does Javier have now?</td>
</tr>
<tr>
<td>Partitive Division</td>
<td>d. There are 8 marbles. 2 friends want to share the marbles so that they each get the same amount. How many marbles can each friend have?</td>
</tr>
<tr>
<td>Compare, Difference Unknown</td>
<td>e. Sarita has 9 toy cars. Her brother Jorge has 6 toy cars. How many more toy cars does Sarita have than Jorge?</td>
</tr>
</tbody>
</table>
Table 2. Selected Post-Assessment Items (English Version)

<table>
<thead>
<tr>
<th>Problem Structure</th>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Join Result Unknown</td>
<td>a. Julio has 6 candies. His sister gives him 6 more candies. How many candies does Julio have now?</td>
</tr>
<tr>
<td>Separate Result Unknown</td>
<td>b. Karla had 13 cookies. She ate 5 of them. How many cookies does Karla have left?</td>
</tr>
<tr>
<td>Join Change Unknown</td>
<td>c. Francisco wants to buy a toy plane that costs 11 dollars. Right now, he only has 7 dollars. How many more dollars does Francisco need so that he can buy the toy plane?</td>
</tr>
<tr>
<td>Multiplication</td>
<td>d. Sara has 3 bags of marbles. There are 6 marbles in each bag. How many marbles does Sara have altogether?</td>
</tr>
<tr>
<td>Partitive Division</td>
<td>e. Estevan had 15 marbles. He shared the marbles with 3 friends so that each friend got the same number of marbles. How many marbles did each friend get? (Estevan did not keep any marbles for himself)</td>
</tr>
<tr>
<td>Measurement Division</td>
<td>f. Alan had 10 cookies, and some little bags. He wants to put 2 cookies in each bag to give to his friends. How many bags can he make?</td>
</tr>
<tr>
<td>Compare</td>
<td>g. Fernando has 12 toy cars. His sister Anabel has 9 toy cars. How many more toy cars does Fernando have than Anabel?</td>
</tr>
<tr>
<td>Multi-Step</td>
<td>h. Javier has 2 bags of candy. There are 4 candies in each bag. Then, Javier gets hungry and eats 3 of the candies. How many candies are left?</td>
</tr>
<tr>
<td>Measurement Division With Remainder</td>
<td>i. In art class, 15 children are going to paint. To paint, they need to sit at tables. Only 4 children can sit at each table. How many tables do they need so that all of the 15 children can paint?</td>
</tr>
</tbody>
</table>
Table 3. Post-Assessment Results: All Three Classrooms

% children correctly solving each problem and using a valid strategy (n=45)

<table>
<thead>
<tr>
<th>Problem Structure</th>
<th>Problem</th>
<th>Correct Answer</th>
<th>Valid Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Join Result</td>
<td>a. Julio has 6 candies. His sister gives him 6 more candies. How many candies does Julio have now?</td>
<td>80%</td>
<td>91%</td>
</tr>
<tr>
<td>Unknown</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Separate Result</td>
<td>b. Karla had 13 cookies. She ate 5 of them. How many cookies does Karla have left?</td>
<td>73%</td>
<td>86%</td>
</tr>
<tr>
<td>Unknown</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Join Change</td>
<td>c. Francisco wants to buy a toy plane that costs 11 dollars. Now, he only has 7 dollars. How many more dollars does Francisco need so that he can buy the toy plane?</td>
<td>56%</td>
<td>69%</td>
</tr>
<tr>
<td>Unknown</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplication</td>
<td>d. Sara has 3 bags of marbles. There are 6 marbles in each bag. How many marbles does Sara have altogether?</td>
<td>49%</td>
<td>64%</td>
</tr>
<tr>
<td>Partitive Division</td>
<td>e. Estevan had 15 marbles. He shared the marbles with 3 friends so that each friend got the same number of marbles. How many marbles did each friend get?</td>
<td>42%</td>
<td>49%</td>
</tr>
<tr>
<td>Division</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement</td>
<td>f. Alan had 10 cookies, and some little bags. He wants to put 2 cookies in each bag to give to his friends. How many bags can he make?</td>
<td>40%</td>
<td>53%</td>
</tr>
<tr>
<td>Division with Remainder</td>
<td>g. Fernando has 12 toy cars. His sister Anabel has 9 toy cars. How many more toy cars does Fernando have than Anabel?</td>
<td>24%</td>
<td>33%</td>
</tr>
<tr>
<td>Compare</td>
<td>h. Javier has 2 bags of candy. There are 4 candies in each bag. Javier gets hungry and eats 3 of the candies. How many candies are left?</td>
<td>44%</td>
<td>49%</td>
</tr>
<tr>
<td>Multi-Step</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement</td>
<td>i. In art class, 15 children are going to paint. To paint, they need to sit at tables. Only 4 children can sit at each table. How many tables do they need so that all the 15 children can paint?</td>
<td>27%</td>
<td>33%</td>
</tr>
</tbody>
</table>
Table 4. Analysis of Variance Across Classroom by Problem Type

<table>
<thead>
<tr>
<th>Problem Type</th>
<th>Significance (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Join Result Unknown</td>
<td>***</td>
</tr>
<tr>
<td>Separate Change Unknown</td>
<td>***</td>
</tr>
<tr>
<td>Join Change Unknown</td>
<td>***</td>
</tr>
<tr>
<td>Multiplication</td>
<td>p = .001</td>
</tr>
<tr>
<td>Partitive Division</td>
<td>p = .001</td>
</tr>
<tr>
<td>Measurement Division</td>
<td>p = .003</td>
</tr>
<tr>
<td>Compare</td>
<td>p &lt; .001</td>
</tr>
<tr>
<td>Multi-Step</td>
<td>*** (p=.06)</td>
</tr>
<tr>
<td>Measurement Division w/ Remainder</td>
<td>*** (p=.054)</td>
</tr>
</tbody>
</table>

1 Contrast analysis of performance of Ms. Arenas’ students as compared to performance of students in the other two classrooms
2 Significance was set at p < .01
3 Indicates an insignificant difference
Table 5. Post-Assessment Results: Ms. Arenas’ Classroom

<table>
<thead>
<tr>
<th>Problem Structure</th>
<th>Ms. Arenas’ Results (n=16)</th>
<th>Carpenter et al. (1993) Results (n=70)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Numbers</td>
<td>Correct Answer</td>
</tr>
<tr>
<td>Join Result Unknown</td>
<td>6, 6</td>
<td>88%</td>
</tr>
<tr>
<td>Separate Result Unknown</td>
<td>13, 5</td>
<td>94%</td>
</tr>
<tr>
<td>Join Change Unknown</td>
<td>7, 11</td>
<td>75%</td>
</tr>
<tr>
<td>Multiplication</td>
<td>3, 6</td>
<td>81%</td>
</tr>
<tr>
<td>Partitive Division</td>
<td>15, 3</td>
<td>75%</td>
</tr>
<tr>
<td>Measurement Division</td>
<td>10, 2</td>
<td>69%</td>
</tr>
<tr>
<td>Compare</td>
<td>9, 12</td>
<td>63%</td>
</tr>
<tr>
<td>Multi-Step</td>
<td>2, 4, 3</td>
<td>63%</td>
</tr>
<tr>
<td>Measurement Division</td>
<td>15, 4</td>
<td>44%</td>
</tr>
</tbody>
</table>
Table 6. Content Analysis of 6 Selected Lessons from each Teacher’s Classroom

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Problem Types(^a)</th>
<th>Aver # of Problems Per Lesson</th>
<th>Number Range(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>JRU(^c)</td>
<td>SRU(^d)</td>
<td>JCU(^e)</td>
</tr>
<tr>
<td>Arenas</td>
<td>7</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Perales</td>
<td>6</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>Field</td>
<td>8</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

\(^a\)Counts represent the number of times teacher presented a given problem type during the six selected lessons.
\(^b\)Represents the range of numbers included in the problems teachers presented.
\(^c\)Join Result Unknown
\(^d\)Separate Result Unknown
\(^e\)Join Change Unknown
\(^f\)Multiplication
\(^g\)Partitive Division
\(^h\)Measurement Division
\(^i\)Compare Difference Unknown
\(^j\)Multi-Step Problem
\(^k\)Represents other problem types. Includes problems such as Separate Change Unknown problems that were not included on post assessment measure.
REFERENCES


Tate, W. (2005). *Access and opportunity to learn are not accidents: Engineering mathematical progress in you school*. The Southeast Eisenhower Regional Consortium for Mathematics and Science at SERVE.


