1 Assignment 1

1. Find the formation tree of \((P \Rightarrow Q) \land (Q \Rightarrow \neg P)) \Rightarrow (P \lor Q)\). What kind of sentence is it?

2. Find the truth value of the preceding sentence when \(P\) has value 0 and \(Q\) has value 1. Also, find the truth value of the preceding sentence when \(P\) has value 0 and \(Q\) has value 0.

3. Find the truth value of \((P \Rightarrow R) \Rightarrow ((P \Rightarrow Q) \land (Q \Rightarrow R)))\) for each of the eight truth valuations.

4. By examining each of the eight truth valuations, prove that it is the case that \(\models ((P \Rightarrow Q) \land (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)\).

5. Is it true that \(\neg (P \land Q) \models \neg P \lor \neg Q\)? Prove that your answer is correct.

6. Is it true that \(\neg (P \land Q) \models \neg P \land \neg Q\)? Prove that your answer is correct.

7. Is it the case that \(P \lor Q, P \Rightarrow \neg R, Q \Rightarrow \neg R\) \(\models \neg R\). Give a proof.

8. Is the following reasoning logically valid. Either I’ll try this problem or I’ll go have a coffee. If I try I won’t get it. But if I go have a coffee then I certainly won’t get it. So I won’t get it.

9. Is it the case that \((P \lor Q) \Rightarrow R, \neg P, \neg Q \models \neg R\). Give a proof.

10. How about this reasoning? If logic is part of mathematics or part of philosophy, then it is a scientific field. But logic is not part of mathematics, neither is it part of philosophy. So it is not a scientific field.

2 Assignment 2

1. Find the formation tree of the sentence \(\forall x (x \text{ rich} \Rightarrow x \text{ happy})\).

2. Find the formation tree of the sentence \(\forall x x \text{ rich} \Rightarrow \forall x x \text{ happy}\).

3. Find an interpretation in which (1) is true and (2) is true.

4. Find an interpretation in which (1) is false and (2) is true.

5. Find an interpretation in which (1) is false and (2) is false.

6. Show that \(\exists x x \text{ rich}, \forall x (x \text{ rich} \Rightarrow x \text{ happy}) \models \exists x x \text{ happy}\) is true.

7. Show that \(\exists x x \text{ happy}, \forall x (x \text{ rich} \Rightarrow x \text{ happy}) \models \exists x x \text{ rich is not true}\).