1. (a) Solve \( \frac{d^2y}{dx^2} + 7 \frac{dy}{dx} + 12y = 0 \).

(b) Deduce a general solution for the equation \( \frac{d^2y}{dx^2} + 7 \frac{dy}{dx} + 12y = xe^{-3x} \).

(c) Show that the particular solution is of the form
\[
y_p = \int^x \left( e^{-3(x-t)} - e^{-4(x-t)} \right) te^{-3t} dt
\]
2. (a) Solve
\[
\begin{align*}
\frac{dx}{dt} - \frac{dy}{dt} + x &= 0 \\
\frac{d^2x}{dt^2} - \frac{dy}{dt} + 3x - y &= 0
\end{align*}
\]
(Hint: \( r = 1 \) is one of the characteristic values).

(b) What is the solution of
\[
\begin{align*}
\frac{dx}{dt} - \frac{dy}{dt} + x &= 2e^t \\
\frac{d^2x}{dt^2} - \frac{dy}{dt} + 3x - y &= 4e^t
\end{align*}
\]
(c) Deduce from part (a) and (b) the general solution of

\[
\begin{aligned}
& t \frac{dx}{dt} - t \frac{dy}{dt} + x = 2t \\
& t(t-1) \frac{d^2x}{dt^2} - t \frac{dy}{dt} + 3x - y = 4t
\end{aligned}
\]

3. (a) Use the transformation \( u = 1/y^2 \) to transform the nonlinear equation \( x^2y' + 2xy - x^2y^3 = 0 \) into a linear one.
(b) Solve the new equation, then deduce the solution to the original problem (a).

4. Solve \((y + y^2 \sin x + 2x \sin y) \, dx - (2y \cos x - x - x^2 \cos y) \, dy = 0\).

5. Consider the equation \(\frac{d^2 y}{dx^2} + \sin y = 0\) (\(*\)). Use the substitution \(p = \frac{dy}{dx}\) and find the equation satisfied by \(p\) (Hint \(\frac{d^2 y}{dx^2} = p \frac{dp}{dy}\)).
(b) Solve for $p$ in terms of $y$.

(c) Deduce that the solutions to our original equation $(\ast)$ is given by the implicit curves

$$x = \pm \int_0^y \frac{dt}{\sqrt{2c_1 + 2 \cos t}} + c_2.$$ 

(d) Extra Credit. What is the implicit equation of the solution when $y(0) = 0$ and $y'(0) = 2$?