Math 125-001
Exam # 1
Partial credit is possible, but you must show all work.

Name: Key

I hereby testify that this is individual work.
Signed:

1. (a) Find a possible formula for the polynomial whose graph is given by:

We have

\[ y = A (x-4)^2 (x-2)(x+3)^2 \]

\[ y(0) = -1 \Rightarrow -1 = A(4)^2(-2)(9) \]

\[ A = \frac{1}{288} \]

(b) Find a possible formula for the rational function whose graph is given by:

\[ y = \frac{A(x+2)^2(x-2)}{(x-1)} \]

\[ y(0) = 5 \Rightarrow 5 = A(4)(-2) \]

\[ A = \frac{5}{8} \]
2. The most authoritative source on the languages of the world lists about 6,800 living languages today. Specialists estimate that at least 40 percent of the languages spoken today would die out by the turn of the century (see http://www.swarthmore.edu/news/text/harrison2.pdf). Assume an exponential model.

(a) Find a formula for the number of languages \( L \) spoken as a function of \( t \), where \( t \) is the number of years starting in 2000.

\[
L = L_0 \left(1 + r\right)^t \\
L = L_0 e^{kt} \\
L = L_0 (0.6)^{t/100}
\]

\[
0.6 = \left(1 + r\right)^{100} \\
(0.6)^{100} = 1 + r \\
r = \left((0.6)^{1/100}\right) - 1 \\
r = 0.005095
\]

\[
L = L_0 (0.994905)^t
\]

(b) What is the "half-life" of the languages of the world?

\[
\frac{1}{2} L_0 = L_0 (0.6)^{t/100}
\]

\[
\ln(0.5) = \frac{t}{100} \ln(0.6) \\
\Rightarrow t = \frac{\ln(0.5)}{\ln(0.6)} (100)
\]

\[
t = 13.57 \text{ years}
\]

(c) When will the peoples of the world speak only 10 languages?

\[
10 = 6800 (0.6)^{t/100}
\]

\[
\ln\left(\frac{10}{6800}\right) = \frac{t}{100} \ln(0.6)
\]

\[
t = \frac{\ln\left(\frac{10}{6800}\right)}{\ln(0.6)} (100) = 1,277 \text{ years}
\]
3. Simplify \( \sin(\arccos(\frac{x}{2})) \).

\[
\cos \theta = \frac{x}{2}
\]

\[
\sin(\theta) = \frac{\sqrt{4-x^2}}{2} = \sqrt{1-\frac{x^2}{4}}.
\]

4. Find the inverse of the function \( y = \frac{e^x - 1}{e^x + 1} \).

\[
y = \frac{e^x - 1}{e^x + 1}
\]

Swap \( x \) and \( y \), then solve for \( y \):

\[
x = \frac{e^y - 1}{e^y + 1}
\]

\[
x e^y + x = e^y - 1
\]

\[
e^y = \frac{-1 - x}{x - 1} = \frac{1 + x}{1 - x}
\]

\[
y = \ln\left(\frac{1 + x}{1 - x}\right)
\]
5. The graph of \( y = f(x) \) is given below. Find the formulas corresponding to the other graphs in terms of \( f \).

\begin{align*}
\text{Graph 1: } \quad y &= f(x) \\
\text{Graph 2: } \quad y &= 2 \int f(x) \\
\text{Graph 3: } \quad y &= f(x) + 1 \\
\text{Graph 4: } \quad y &= f(x + 0.5)
\end{align*}
6. A population of animals oscillates sinusoidally between a low of 700 on January 1 and a high of 900 on July 1.

(a) Find a formula for the population as a function of time, \( t \), measured in months starting on January 1.

\[
T = 12 \text{ months} \Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{12} = \frac{\pi}{6}
\]

\[
y = 800 - 100 \cos \left( \frac{\pi t}{6} \right) \\
\text{or} \quad y = 800 + 100 \sin \frac{\pi}{6} (t - 3)
\]

(b) What is the size of the population on October 1?

\[
t = 9 \Rightarrow y = 800 - 100 \cos \left( \frac{\pi \cdot 9}{6} \right) = 800
\]

(c) When does the population reach the size of 850?

\[
y = 850 \Rightarrow 850 = 800 - 100 \cos \left( \frac{\pi t}{6} \right)
\]

\[
\cos \left( \frac{\pi t}{6} \right) = \frac{-50}{100} = -\frac{1}{2}
\]

\[
\cos \left( \frac{\pi t}{6} \right) = \frac{1}{2}
\]

\[
\frac{\pi t}{6} = \cos^{-1} \left( \frac{1}{2} \right) = \frac{2\pi}{3} \quad \Rightarrow \quad t = 4 \text{ or } 8
\]

\[
\text{or} \quad \pi + \frac{\pi}{3} = \frac{4\pi}{3}
\]