1. (a) For what values of \( n \) in the field \( \mathbb{Z}_7 \) is \( 3n^3 - 4n^2 + 2n - 5 = 0 \)?

(b) Solve in the field \( \mathbb{Z}_7 \) the equation \( n! + 1 = 0 \)
2. (a) Find the multiplicative inverse of 18 in the field \( \mathbb{Z}_{41} \) (note that 41 is a prime number).

(b) Find all the \( x \) and \( y \) in \( \mathbb{Z} \) for which

\[
18x + 41y = 1.
\]
3. Let $a > 1$ be a given fixed real number. Define the sequence $(x_n)_{n \in \mathbb{N}}$ by: 
$x_1 = 1$ and $x_{n+1} = \sqrt{a + x_n}$

(a) Show that $(x_n)_{n \in \mathbb{N}}$ is a monotone increasing sequence.

(b) Show that for all $n \in \mathbb{N}^*$, $\sqrt{a} < x_n < 2a$.

(c) Deduce that $(x_n)_{n \in \mathbb{N}}$ is convergent. What is its limit?
4. Consider the sequence $x_1 = 0; \ x_{n+1} = \frac{3}{\sqrt{6 - x_n^2}}.

(a) Show that for $n \geq 1$, $0 \leq x_n \leq \sqrt{3}$

(b) Show that $x_{n+1} > x_n$ for all $n$.

(c) Show that $(x_n)_{n \geq 1}$ is a convergent sequence. What is its limit?
5. Consider the sequence \( x_1 = 0; x_{n+1} = \frac{3}{\sqrt{6 - x_n^2}} \).

(a) Let \( y_n = \frac{x_n^2}{3 - x_n^2} \). Show that \( y_{n+1} - y_n = 1 \).

(b) Use the result in part (a) to find explicit formulas for \( x_n \) and \( y_n \).

(c) Use your explicit form from part (b) to find \( \lim_{n \to \infty} x_n \).
6. (a) Show that \( x_n = \frac{\sin n}{2^n} \) is a Cauchy sequence, while \( y_n = (-1)^n + \frac{1}{n} \) is not.

(b) Show that \( x_n = 1 + \frac{\cos 1}{2^1} + \frac{\cos 2}{2^2} + \ldots + \frac{\cos n}{2^n} \) is a Cauchy sequence.