1. Determine whether the following series are convergent or divergent. Justify your answer by stating which test you used in each case.

(a) \( \sum_{n=1}^{\infty} \frac{n^3}{2^n} \).

(b) \( \sum_{n=1}^{\infty} \frac{2^n}{n!} \).

(c) \( \sum_{n=1}^{\infty} \frac{3n^2 - 5n + 7}{n^4 - 3n^2 - 3n - 1} \).

(d) \( \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} \).

(e) \( \sum_{n=1}^{\infty} \frac{n!}{\sqrt{n(n+1)}} \).
2. Determine whether the following series conditionally convergent, absolutely convergent or divergent? Justify your answer in each case.

(a) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\ln(n+1)}. \]

(b) \[ \sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 1}. \]

(c) \[ \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}. \]

(d) \[ \sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}. \]

3. Prove the following ratio test: If \((a_n)_{n\in\mathbb{N}}\) and \((b_n)_{n\in\mathbb{N}}\) are positive sequences such that \[ \sum_{n=1}^{\infty} b_n \] converges and \[ \frac{a_{n+1}}{a_n} \leq \frac{b_{n+1}}{b_n}. \] Then \[ \sum_{n=1}^{\infty} a_n \] converges.
4. For what values of \( p \) is the series
\[
\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}
\]
convergent? Justify your answer.

5. Determine whether the following functions converge uniformly on the indicated interval. Justify your answer.

(a) \( f_n(x) = \frac{x^n}{n} \), for \( x \in [-1, 1] \).

(b) \( f_n(x) = \frac{x}{x+n} \), for \( x \in [0, a] \), and \( a > 0 \) a fixed real number.

(c) \( f_n(x) = \frac{x}{x+n} \), for \( x \in [0, \infty) \).
(d) \( f_n(x) = \frac{nx}{1+nx} \), for \( x \in [a, \infty) \) and \( a > 0 \) a fixed real number.

(e) \( f_n(x) = \frac{nx}{1+nx^{1/n}} \), for \( x \in [0, \infty) \).

6. Let \( f_n(x) = nx^n(1-x) \) for \( x \in [0,1] \).

(a) Find \( f(x) = \lim_{n \to \infty} f_n(x) \).

(b) Show that the convergence of this sequence of functions is not uniform on \([0,1]\).

(c) Does \( \int_0^1 f(x)dx = \lim_{n \to \infty} \int_0^1 f_n(x)dx \)? Justify your answer.

(d) Does \( f'(x) = \lim_{n \to \infty} f'_n(x) \)? Justify your answer.
7. Use the Weirstrass M-Test (Chapter 9, p. 296) to determine whether the following Power series are uniformly convergent. Justify your answer.

(a) \( \sum_{n=1}^{\infty} \frac{x^2}{n^2} \) for \( x \in [0, 5] \).

(b) \( \sum_{n=1}^{\infty} \frac{x^2}{n^2} \) for \( x \in [5, \infty) \).

(c) \( \sum_{n=1}^{\infty} \frac{1}{1 + n^2 x^2} \) for \( x \in [1, \infty) \).

(d) \( \sum_{n=1}^{\infty} \frac{x^{2n}}{(n + x)^2} \) for \( x \in [0, 1] \).

(e) \( \sum_{n=1}^{\infty} \frac{1}{n^x} \) for \( x \in [\sqrt{2}, \infty) \).
8. Find the radius and interval of convergence and study the (absolute, conditional) convergence of each of the series.

(a) \( \sum_{n=1}^{\infty} \frac{x^n}{n} \).

(b) \( \sum_{n=1}^{\infty} \frac{n}{3^n} (x - 2)^n \).

(c) \( \sum_{n=1}^{\infty} \frac{n^3}{2^n} (x - 1)^{2n} \).
9. (a) Show that
\[
\int_0^x \arctan t\,dt = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^{2n}}{2n(2n-1)}
\]
for \(|x| < 1\).

(b) Show that the formula in part (a) is valid for \(x = 1\).

(c) Assuming the series \(1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{6} - \frac{1}{7} + \cdots\) is convergent, find its each value using parts (a) and (b).