1. (5 points) Warfarin is a drug used as an anticoagulant. After administration of the drug is stopped, the quantity remaining in a patient’s body decreases at a rate proportional to the quantity remaining. The half-life of warfarin in the body is 40 hours.

a. Sketch the quantity of Warfarin in a patient’s body \( Q \) as a function of the time \( t \) since stopping administration of the drug. Mark the 40 hours on your graph.

b. Write a differential equation satisfied by \( Q \).

We are given that the rate of change \( \frac{dQ}{dt} \) is decreasing proportional \((-k)\) to the amount remaining in the body \( Q \). Therefore, we can construct our differential equation:

\[
\frac{dQ}{dt} = -kQ
\]

By using the technique of separation of variables, we can now find a general solution for \( Q \).

\[
\frac{dQ}{Q} = (-k)dt \Rightarrow \ln|Q| = -kt + C \Rightarrow Q(t) = Ce^{-kt}
\]

Since \( Q(0) \) is the initial amount of the drug in the body, we denote the constant, \( C \), as \( Q_o \). Now, we can solve for the decay constant, \( k \), by using the given information on the drug’s half-life.

\[
Q(40) = \frac{1}{2}Q_o = Q_o e^{-40k} \Rightarrow \frac{1}{2} = e^{-40k}
\]
\[ \Rightarrow \ln\left(\frac{1}{2}\right) = \ln(1) - \ln(2) = -\ln(2) = -40k \]
\[ \Rightarrow k = \frac{\ln(2)}{40} \]

2. (5 points) Solve the following integral. Use differentiation to check your answer.

\[ \int \left( \cos(\phi) + \frac{1}{\cos^2(\phi)} \right) d\phi \]

The first step is to rewrite \( \frac{1}{\cos^2(\phi)} \) as \( \sec^2(\phi) \). Now, we recognize that \( \sec^2(\phi) \) is the derivative of \( \tan(\phi) \). Therefore:

\[ \int \left( \cos(\phi) + \frac{1}{\cos^2(\phi)} \right) d\phi = \sin(\phi) + \tan(\phi) + C \]

It is also important to remember the constant of integration in our solution.

To check our work, we differentiate each term of our solution.

\[ \frac{d}{d\phi} [\sin(\phi) + \tan(\phi) + C] = \cos(\phi) + \frac{\cos^2(\phi) + \sin^2(\phi)}{\cos^2(\phi)} \]
\[ = \cos(\phi) + \frac{1}{\cos^2(\phi)} \]