Introduction to Quadratic Functions

Victor I. Piercey

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Motivation

Why does one go into business?

What is the goal of a person running a business?

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A quadratic function is a function of the form

\[ f(x) = ax^2 + bx + c \]

for some fixed numbers \( a, b, c \) with \( a \neq 0 \).

For example: \( f(x) = 2x^2 - 12x + 10 \) is a quadratic function.
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Every quadratic function can be written in standard form:

\[ f(x) = a(x - h)^2 + k. \]

The goals today are:

1. Figure out how to graph a quadratic function written in standard form by hand, and

2. Figure out how to write a given quadratic function written as \( f(x) = ax^2 + bx + c \) in standard form.
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2. figure out how to write a given quadratic function written as \( f(x) = ax^2 + bx + c \) in standard form.
A quadratic function in standard form $y = a(x - h)^2 + k$ can be realized as a sequence of transformations of the base graph $y = x^2$. Use transformations of the graph $y = x^2$ to graph the following quadratic functions:

1. $f(x) = x^2 - 3$
2. $f(x) = -(x - 2)^2$
3. $f(x) = 3(x + 1)^2 - 2$
4. $f(x) = -2(x - 1)^2 + 1$. 
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Discussion: Features of Quadratic Functions

For the functions that you graphed, determine:

1. Domain

2. Range

3. Intercepts

4. Max/Min?

5. Symmetry?
Given some quadratic function \( f(x) = ax^2 + bx + c \), how can we rewrite the function in standard form?

Typically one would complete the square.

Another way to do this uses the axis of symmetry. First we will do this for the function

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and then we will do it in general.
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Finding Standard Form

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How to Find Standard Form

Bottom line, using the axis of symmetry, the quadratic function $f(x) = ax^2 + bx + c$ is the same as

$$y = a \left( x + \frac{b}{2a} \right)^2 + f \left( \frac{b}{2a} \right).$$