Math 125, Fall 2015
Instructor: Nikola Kamburov

Practice Exam 2

Your Name (Please Print): ____________________________________________

Signature: __________________________________________________________

Directions:
- Do not open this exam until you are told to do so.
- The exam is closed book/notes. All electronic devices that can receive or transmit a wireless signal must be turned off during the exam. Calculators may not be shared.
- Remember: Unless stated otherwise, answers without explanations will not count! You should show your work. Solve each problem in the space provided. If you need more space you can attach an extra page to your exam paper, but make a note you did so. If you use any named theorem, cite it.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Score</th>
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<tbody>
<tr>
<td>1</td>
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Problem 1. (20pts)

(a) Mark whether the statement is true or false. (you do not have provide an explanation) (10)

- If $f''(x) > 0$ then $f(x)$ is increasing. \text{T \ F}
- The fifth derivative of $2^x$ is $2^x (\ln 2)^5$. \text{T \ F}
- If $f(x)$ is continuous on $(0, 3)$ and $f(1) = f(2) = 0$, then $f(c) = 0$ for some $1 < c < 2$. \text{T \ F}
- The seventh derivative of $\sin x$ is $\cos x$. \text{T \ F}
- If $f'(x)f''(x) < 0$ then $f'(x)^2$ is decreasing. \text{T \ F}

(b) Given the graph $y = f(x)$ below, determine the value(s) of $x$ in the domain of $f$, at which

- $f$ has a discontinuity; (3)

- $f$ is continuous but not differentiable; (4)

- $f$ is differentiable but not continuous. (3)
Problem 2. Given the information about the differentiable functions $f(x)$ and $g(x)$ in the table below, answer the following questions.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
<th>$f'(x)$</th>
<th>$g'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>-3</td>
<td>5</td>
</tr>
<tr>
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<td>3</td>
<td>-1</td>
<td>-2</td>
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</table>

(a) Find the equation of the tangent line to the graph of $f$ at the point $(1, 2)$; 

(b) Compute $h'(2)$ if $h(x) = f(x)/g(x)$; 

(c) Compute $h'(1)$ if $h(x) = g(f(x))$; 

(d) Find the equation of the tangent line to the graph $y = h(x)$ at $x = 4$ if 
    $h(x) = g(\sqrt{x}) + (f(x - 3))^2$. 


Problem 3. The graph of $f'(x)$, the derivative of $f(x)$, is given below.

(a) On which intervals is $f$ increasing? Select all that apply.

A. $(0, x_1)$  
B. $(x_1, x_2)$  
C. $(x_2, x_3)$  
D. $(x_3, x_4)$  
E. $(x_4, x_5)$

(b) At which point(s) does $f$ attain a local maximum?

A. $x_1$  
B. $x_2$  
C. $x_3$  
D. $x_4$

(c) On which intervals is $f$ concave down? Select all that apply.

A. $(0, x_1)$  
B. $(x_1, x_2)$  
C. $(x_2, x_3)$  
D. $(x_3, x_4)$  
E. $(x_4, x_5)$

Problem 4. Let $Q(T)$ be the amount of ice-cream, in thousands of pints, sold on a single day in Tucson when the average daily temperature is $T$ degrees Fahrenheit.

(a) Give a practical interpretation of $Q'(90) = 4$. Include units.

(b) Assuming that $Q(T)$ is an increasing function of $T$, give a practical interpretation of the statements $Q^{-1}(92.3) = 90$ and $[Q^{-1}]'(92.3) = 0.25$.

(c) Estimate the average temperature on the day when 89,900 pints of ice-cream were sold.
Problem 5. Let $a > 1$. Show that the polynomial $p(x) = ax^4 - 2ax + a - 1$ has a root in $[1, 1]$.

Problem 6. Compute the limits if they exist. If not, argue why.

(a) $\lim_{x \to \infty} \frac{x^3 + 2x}{2x^2 - 3x^3 + 1}$

(b) $\lim_{x \to 2} \frac{x^2|x - 2|}{x - 2}$

(c) $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$