Homework 1A
Math 456/556

Directions: answer using complete sentences. Write each problem on a separate sheet of paper.

**Question 1** Find all values of a and b so that \( u(x, y) = \sin(ax) \exp(by) \) a solution of Laplace’s equation
\[
 u_{xx} + u_{yy} = 0.
\]

**Question 2** The equation \( u_t = -\Delta \Delta u + u^2 \) is a conservation law of the form \( u_t + \nabla \cdot J = Q \). Identify the flux \( J \) and source term \( Q \).

**Question 3** A population of microorganisms has a density \( \rho(x, t) \) cells per unit area (you may suppose \( x \in \mathbb{R}^2 \)). They seek out oxygen (“oxytaxis”) by moving at a velocity proportional to the gradient of oxygen concentration \( c(x) \). As this occurs, cells divide, causing the population to grow (at each point in space) at a rate proportional to \( \rho \). Suppose all this happens in a bounded domain \( \Omega \), and no cells can pass through the boundary. Write down a PDE and boundary condition for the function \( \rho \). For simplicity, you may set the constants of proportionality equal to one.

**Question 4** We are not prepared to solve problem
\[
 u_t = uu_x + u^2, \quad u(0, t) = 1,
\]
on the domain \( \{ x > 0, t > 0 \} \). A steady state solution, however, can be obtained since it solves an ODE. Find this solution which also satisfies the stated boundary condition.

**Question 5** Are the following linear equations? If they are, put them in the form \( Lu = f \), identify the linear operator \( L \), and classify them as homogeneous or inhomogeneous:
\[
 u_t = (xu_x)_x + (yu_y)_y + xy, \quad uu_x + u_{xxx} = u_t, \quad u_{tt} = u_{xxxx} + u_{yyyy}.
\]

**Question 6** Check that \( u_p = -\frac{1}{4} \sin(2t) \) is a particular solution to the equation
\[
 u_{tt} = c^2 u_{xx} + \sin(2t)
\]
and that \( w_n(x, t) = \sin(nx) \sin(cnt) \) is a solution of the wave equation
\[
 w_{tt} = c^2 w_{xx}.
\]
for \( n = 1, 2, 3 \). What is the most general solution to (1) that can be formed from \( u_p \) and \( w_1, w_2, w_3 \)?