1. Find an asymptotic expansion with two non-zero terms for solutions of \( \cos x = \frac{x}{\epsilon} \). Check your approximation for \( \epsilon = 0.3 \) against a numerical solution (obtained by, for example, plotting \( \cos \) against \( -x/\epsilon \)).

2. Find a three term asymptotic expansion of \( \ln(1 + e^{\epsilon^{-1}}) \) for \( \epsilon \to 0 \). Explain why it is an asymptotic series.

3. Consider \( \epsilon x^8 - \epsilon^2 x^6 + x - 2 = 0 \) for small \( \epsilon \). Investigate dominant balance by considering roots of the form \( x \sim \epsilon^{-n} \). Then find a two-term expansion for all solutions.

4. Find an expansion for the large roots of \( x \tan(x) = 1 \) by first writing as iteration \( x_{n+1} = k\pi + \tan^{-1}(1/x_n) \), where \( k \) is a large integer.

5. Let \( A, B \) be nonsingular \( n \times n \) matrices. Find a three term expansion of \( (A + \epsilon B)^{-1} \) for \( \epsilon \to 0 \).
(Hint: let \( C \) be equal to the inverse so \( C(A + \epsilon B) = I \))

6. Find a two-term expansion for \( O(1) \) eigenvalues of the Sturm-Liouville problem
\[
 u''(x) + \lambda u = 0, \quad u(0) = 0, \quad u(L) = \epsilon u'(L).
\]
Explain why your answer is not valid when \( \lambda = O(\epsilon^{-2}) \) (hint: what terms are dominant in the boundary condition?)