Homework 1
Math 587

1. Find an asymptotic expansion (to several terms) of \( \ln(1 + e^{\epsilon^{-1}}) \) for \( \epsilon \to 0 \).
(Hint: factor out the dominant term inside the log first)

2. Find an asymptotic expansion with two non-zero terms for solutions of \( \cos x = x/\epsilon \). Check your approximation for \( \epsilon = 0.3 \) against a numerical solution (obtained by, for example, plotting \( \cos \) against \( -x/\epsilon \)).

3. Consider \( \epsilon x^5 - \epsilon^2 x^6 + x - 2 = 0 \) for small \( \epsilon \). Investigate dominant balance by considering roots of the form \( x \sim \epsilon^{-\alpha} \). Then find a two-term expansion for all solutions.

4. Find the large roots of \( x \tan(x) = 1 \). (Hint: write as iteration \( x_{n+1} = k \pi + \tan^{-1}(1/x_n) \) where \( k \gg 1 \) and \( \tan^{-1} \) is the principal branch).

5. Let \( A, B \) be nonsingular \( n \times n \) matrices. Find a three term expansion of \( (A + \epsilon B)^{-1} \) for \( \epsilon \to 0 \).
(Hint: let \( C \) be equal to the inverse so \( C(A + \epsilon B) = I \))