Math 468 / 568 Homework #10
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Due Tuesday 4/13

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Instructions:

• I encourage you to work together, but you must write up your own answers.

Problems

1. HPS 3.16

2. HPS 3.21

3. Consider a Markov jump process $X(t)$ with state space $S = \{0, 1, 2, \ldots\}$ and transition probabilities

$$Q_{xy} = \begin{cases} 
1, & x = 0 \text{ and } y = 1 \\
p, & x > 0 \text{ and } y = x + 1 \\
q, & x > 0 \text{ and } y = 0 \\
0, & \text{otherwise}
\end{cases}$$

Assume $0 < p < 1$, $q = 1 - p$, and $q_x > 0$ for all $x$.

(a) Draw the state diagram for the embedded chain. Include all transition probabilities.

(b) Is $X(t)$ irreducible? Why?

(c) Assuming $X(t)$ has a stationary distribution $\pi$, express $\pi(x)$ in terms of $\pi(0)$.

(d) Give a necessary and sufficient condition for $X(t)$ to have a stationary distribution.

(e) Let $q_x = 1/(1 + x)$ for $x = 0, 1, 2, \ldots$. Find $E_x(T_x)$.  
