The following problems are related to Problem 42 in Section 7.6 of the textbook.

Use the following notation (assume $\alpha > 0$, as in the textbook):

$$F_0(\alpha) = \int_0^\infty \frac{e^{-y/\alpha}}{\alpha} \, dy, \quad F_1(\alpha) = \int_0^\infty \frac{ye^{-y/\alpha}}{\alpha} \, dy, \quad F_2(\alpha) = \int_0^\infty \frac{y^2e^{-y/\alpha}}{\alpha} \, dy.$$  

Now do the following problems. In the end, you will have a solution of Problem 42.

A.1. Do one integration by parts to express $\int_0^b \frac{ye^{-y/\alpha}}{\alpha} \, dy$ in terms of $\int_0^b \frac{e^{-y/\alpha}}{\alpha} \, dy$.  

(Do not do the last integral yet; stop when you have expressed the first integral in terms of the second.)

A.2. Take a limit, $\lim_{b \to \infty}$, to express $F_1(\alpha)$ in terms of $F_0(\alpha)$. (No integration symbols in your final answer; just write a formula for $F_1(\alpha)$ in terms of $F_0(\alpha)$.)

A.3. Similarly, do one integration by parts to express $\int_0^b \frac{y^2e^{-y/\alpha}}{\alpha} \, dy$ in terms of $\int_0^b \frac{ye^{-y/\alpha}}{\alpha} \, dy$.

A.4. Take a limit, $\lim_{b \to \infty}$, and do some simple, minor algebra to express $F_2(\alpha)$ in terms of $F_1(\alpha)$.

You now should have formulas expressing $F_2(\alpha)$ in terms of $F_1(\alpha)$ and $F_1(\alpha)$ in terms of $F_0(\alpha)$.

A.5. Use substitution to express $F_0(\alpha) = \int_0^\infty \frac{e^{-y/\alpha}}{\alpha} \, dy$ in terms of $\int_0^\infty e^{-x} \, dx$. (Don’t do the last integral yet.)

A.6. Calculate the improper integral $\int_0^\infty e^{-x} \, dx$ and hence use Problem 5 to determine $F_0(\alpha)$.

A.7. Now use the results of Problem 2 and 4 to find $F_1(\alpha)$ and $F_2(\alpha)$.

Note that you have now solved Problem 42 in Section 7.6.

A.8. For more practice and to make sure you know what you’re doing, use a similar approach to find $\int_0^\infty \frac{y^3e^{-y/\alpha}}{\alpha} \, dy$ (do one integration by parts to get a formula for this in terms of $F_2(\alpha)$.)