Math 129-1  Exam 1 COMMENTS (regarding one version of the exam), Problems 1-2, 8

None of these problems requires a calculator for doing the integration. In all cases, in order to receive full credit you should show enough work that it is clear that you can do the problem correctly and completely without using a calculator.

1. (15 points) For the following integral, choose an appropriate trigonometric substitution (or an appropriate hyperbolic substitution), and make the substitution completely in the integral. After completing the substitution (so that there is no variable \( x \) in the integral), IT IS NOT NECESSARY TO SIMPLIFY OR TO FIND THE ANTIDERIVATIVE.

\[
\int \frac{1}{x^2 \sqrt{10 + x^2}} \, dx.
\]

COMMENT. Read The Instructions:
After completing the substitution (so that there is no variable \( x \) in the integral), IT IS NOT NECESSARY TO SIMPLIFY OR TO FIND THE ANTIDERIVATIVE.

2. (20 points) Find \( \int x e^{ax} \, dx \) (where \( a \) is a constant) without using tables or a calculator.

COMMENT. This is an “indefinite integral” – an antiderivative. You can easily CHECK, and you should if you have time.

3. (25 points) Find the exact area under the curve \( y = (t + 1) e^{(5t+5)} \) between \( t = 0 \) and \( t = 1 \). Do not use tables or a calculator.

COMMENT. USE PROBLEM 2!
We want \( \int (t + 1) e^{5t+5} \, dt \), and the substitution \( x = t + 1 \) changes this to \( \int x e^{5x} \, dx \).
This is exactly the same form as the integral we did in Problem 2; THERE IS NO NEED TO DO IT AGAIN.

4. (15 points) Show how to expand in partial fractions (i.e., split into partial fractions). You don’t have to do any calculations. Just show what a partial fraction expansion would look like.

\[
\frac{x}{(x^2 + 9)(x - 3)^2}.
\]

COMMENT. Read The Instructions:
You don’t have to do any calculations. Just show what a partial fraction expansion would look like.

5. (25 points) Evaluate the following integral using partial fractions, without using tables. \( \int \frac{x^2}{x^2 - 25} \, dx \)

COMMENTS on DOING the problem. This is similar to, but much simpler than, a homework exercise from 7.4., which was assigned for homework, done in class, and has a solution posted online (for the more difficult problem assigned as homework). Be sure to be familiar with the outline of how to do such integrals given in the box following Example 5.

COMMENTS on CHECKING your answer. The result of splitting a rational function into partial fractions can ALWAYS be checked by adding your answer (common denominator) and checking. And since this is an antiderivative, it can be checked in the usual way.
6. (15 points) Evaluate the integral \( \int \frac{1}{\sqrt{10 + x^2}} \, dx \) by using the substitution
\[
u = x + \sqrt{10 + x^2} \]

COMMENT. This was done in class.

7. (25 points) Calculate the following approximations for the definite integral \( \int_{0}^{\pi/2} \sin x \, dx \) without using a calculator.

COMMENT. The solution is based on the function, and the graph of the function, that you’re integrating; for this problem, that is the \( \sin \). You don’t DO the integral and THEN do an approximation.

8. (25 points) Using simple algebra, solve for \( x \):
\[
e^{2x} - 4e^{x} + 4 = 0.\]

COMMENT. This is basic algebra, using the quadratic formula and the fact that \( e^{2x} = (e^{x})^2 \).