7. Calculate the following approximations for the definite integral \( \int_0^{\pi/2} \sin x \, dx \) without using a calculator.

a. **Find**, without using a program on a calculator, \( \text{RIGHT}(2) \) (the right hand sum with \( n = 2 \)). **Illustrate** on a graph and **explain** whether the approximation is an overestimate or and underestimate of the actual integral.

b. **Illustrate** \( \text{TRAP}(2) \) (the trapezoidal approximation with \( n = 2 \)) on a graph and **explain** from the graph whether the approximation is an overestimate or an underestimate of the actual integral. (No calculations necessary.)

**SOLUTIONS.** Since both problems have \( n = 2 \), we divide the interval of integration, \([0, \pi/2] \), into two equal parts, using the points \( x_0 = 0, \ x_1 = \pi/4, \) and \( x_2 = \pi/2 \). Then the length of each interval is \( \Delta x = \pi/4 \), so

\[
\text{RIGHT}(2) = \sin(\pi/4)(\pi/4) + \sin(\pi/2)(\pi/4) = [\sin(\pi/4) + \sin(\pi/2)](\pi/4) = [\sqrt{2}/2 + 1](\pi/4).
\]

By looking at the graph – not shown here – we see that the right-hand sum is an **overestimate** (this can also be concluded from the fact that the \( \sin \) function is increasing on this interval, but you’re asked to use the graph).

b. By looking at the graph – not shown here – we see that \( \text{TRAP}(2) \) is an **underestimate** (this can also be concluded from the fact that the \( \sin \) function is concave down on this interval), but you’re asked to use the graph.)

8. See a separate page for the solution of Problem 8.