6. **(20 pts)** This problem is independent of the preceding. Using strips or slices as we have done in this class, find exactly the volume generated when the region in the first quadrant bounded by \( y = x^2 \) and \( y = 2 \) is rotated around the \( y \)-axis.

**SOLUTION.** You should draw a sketch, showing the region bounded by the given curves and a horizontal slice which goes from the \( y \)-axis to the curve \( y = x^2 \). The radius of the “coin” or “disk” obtained when this is rotated around the \( y \)-axis is

\[ x_{\text{right}} - x_{\text{left}} = \sqrt{y} - 0 = \sqrt{y}, \]

so the volume of the disk is \( \Delta V = \pi (\sqrt{y})^2 \Delta y = \pi y \Delta y \).

The total volume is

\[ \int_0^2 \pi y \, dy = 2\pi. \]

7. **(20 pts)** For \( x > 0 \), determine if the following are geometric series, and (of course) explain your answer. If convergent, find the sum.

\[ \text{a. } 1 + e^x + e^{2x} + e^{3x} + ... \quad \text{b. } 1 + e^{-x} + e^{-2x} + e^{-3x} + ... \]

**SOLUTION.** The first sum can be written as

\[ 1 + (e^x) + (e^x)^2 + (e^x)^3 + ... = \sum_{n=0}^{\infty} \left( e^x \right)^n \]

So this is a geometric series with common ratio \( e^x > 1 \). This it is divergent.

The second series is the same as the first with \( x \) replaced by \( -x \) (or, equivalently, \( e^x \) replaced by \( e^{-x} \)). It is a geometric series with common ratio \( e^{-x} \), with \( 0 < e^{-x} < 1 \), so it is convergent.

\[ 1 + (e^{-x}) + (e^{-x})^2 + (e^{-x})^3 + ... = \sum_{n=0}^{\infty} \left( e^{-x} \right)^n = \frac{1}{1 - e^{-x}} \]

**COMMENT.** Read The Instructions. You are asked to find the sum if the series is convergent.

8. **(15 pts)** Find the limit of the following sequence:

\[ \frac{5n + 6}{2n - 4} \]

**SOLUTION.** The sequence converges to \( 5/2 \) because when \( n \) is large, the 6 in the numerator and -4 in the denominator are irrelevant. More precisely,

\[ \lim_{n \to \infty} \frac{5n + 6}{2n - 4} = \lim_{n \to \infty} \frac{5 + \frac{6}{n}}{2 - \frac{4}{n}} = \frac{5 + 0}{2 - 0} = \frac{5}{2}. \]

**COMMENT.** We are using the simple algebraic fact that

\[ \frac{5n + 6}{2n - 4} = \frac{5 + 6/n}{2 - 4/n}. \]

This is NOT the same as the false statement

\[ \frac{5n + 6}{2n - 4} = \frac{5}{2} + \frac{6/n}{-4/n}. \] Furthermore, this last fraction does NOT approach \( \frac{5}{2} + 0 \).