Before writing your answers to these questions, THINK about the following:
There is – or there is not – room on this sheet of paper to neatly write your solutions.
(Sometimes there will be, sometimes not.)

Background. This is the latest in a series of worksheets discussing how one proves a “theorem” of the form $\chi_A^{-1}(Q) = P$, where $A$ is a given set, $Q$ is a given set of real numbers or a given real number (it was 1 in the latest worksheets), and $P$ is presumably a set that YOU have discovered which you think is equal to the given pre-image.

There are several “layers” to this – not STEPS, but LAYERS. The point of the worksheets is to help you think through the layers and use simply logic and definitions to do the steps.

- Zooming all the way out, the “outside layer” is that you are trying to prove that two sets are equal. You should, by now, know how to do this and know how to start each part of the proof.
- Zooming in a little, the “next layer” is that one of the sets is a pre-image. So at this layer, the focus is on the definition of pre-image, what it means for something to be an element of a pre-image, and (eventually) how you prove that something is in a pre-image.
- Finally, the “inside layer” in this example is the fact that the pre-image happens to involve the characteristic function of the set $A$, so you need to know the definition of characteristic function.

Many students fail to keep this order in mind and get all tangled up, just like you would if you tried to take off your socks before you take off your shoes, or try to memorize “steps” instead of understanding the layers and UNDERSTANDING the logic of the resulting steps.

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Now, do this. Again, we are given sets $A$ and $Q$ and we have found a set $P$. Our ultimate goal would be to prove that $\chi_A^{-1}(Q) = P$. We are focusing only on the beginning of one part of this proof.

3. Let’s think about how we prove that every element of $\chi_A^{-1}(Q)$ is in $P$. In the step numbered 2 below, we will use the definition of the pre-image.

Write the following as indicated:

0. Write this: “Proof. First, we will prove that ... [complete the sentence].”

1. Give a reasonable first sentence, as a start of this proof.

2. Next, USE THE DEFINITION OF PRE-IMAGE (not the definition of characteristic function) to draw a conclusion.

3. Now, USE THE DEFINITION OF THE SPECIFIC FUNCTION $\chi_A$ (not the definition of pre-image) to draw a conclusion.

So far, the set $P$ should not have appeared in your argument in steps 1, 2, and 3.

By the way, I numbered the steps above for convenience. You should NOT number the steps in your proof. The point of this is to get you to THINK about how to do a proof, not to give you a template to copy when doing a proof.