1. Let $a$ be a real number and let $\mathcal{B} = \{ (\infty, x] : x \text{ is real and } x > a \}$. Give the proofs requested below based on the standard approach to proving that two given sets are equal and on the definition of union and intersection.

(a) Find $\bigcup \mathcal{B}$ and prove your answer.

[The following is just to explain the notation. This notation is not necessary for doing problem (a).]

$\bigcup \mathcal{B}$ is also written $\bigcup_{B \in \mathcal{B}} B$ (in the textbook) and $\bigcup_{x > a} (\infty, x]$ (by many mathematicians).

(b) Find $\bigcap \mathcal{B}$ and prove your answer.

2. Let $\mathcal{C}$ be a collection of sets. Find the union and intersection as indicated below, and prove your answer.

(a) Suppose there is a set $B$ in $\mathcal{C}$ such that every set in $\mathcal{C}$ is a subset of $B$. Find $\bigcup \mathcal{C}$ and prove your answer.

(b) Suppose there is a set $S$ in $\mathcal{C}$ which is a subset of every set in $\mathcal{C}$. Find $\bigcap \mathcal{C}$ and prove your answer.

3. (a) Is the set $(-\infty, \infty)$ an element of the collection $\mathcal{B}$ in Problem 1 above?

(b) Is the set $(-\infty, 3]$ an element of the collection $\mathcal{B}$ in Problem 1 above?