Regarding Problem 15.1.14.

The point of the first couple of problems below is the following: Time can be saved by thinking about the relation between what has been done before and what is to be done now, which in this problem was: The full Fourier series on the original interval \([-\pi, \pi]\), and the cosine, sine, and full Fourier series on the half-interval \([0, \pi]\). And that work itself can be cut in half by thinking about things.

We use \(g\) as the name of the function dealt with in Problem 15.1.14 to distinguish it from the function in Problem 15.1.11 (both functions are \(f\) in the textbook). It is given on the interval \([-\pi, \pi]\).

We will use \(g_p\) to denote the function on \([-\pi, \pi]\) obtained as follows: Restrict the original \(g\) to the half-interval and then construct the periodic extension \([\text{period } \pi]\) to the interval \([-\pi, \pi]\) (and beyond, if desired). (You can also use the notations \(g_o\) and \(g_e\) as we did for \(f\), if useful.)

1. Is the function \(g\) even or odd (or neither)? If so, what does this tell you about the coefficients in the Fourier series you are asked to obtain on the interval \([-\pi, \pi]\)?

2. Continuing with the thought of Problem 1 just above, what does this tell you about the cosine series or the sine series requested on the interval \([0, \pi]\)?

3. Sketch graphs of the function \(f\) from Problem 15.1.11 on the interval \([-\pi, \pi]\), and of the function \(g_p\) (see the definition above) for this problem on the interval \([-\pi/2, \pi/2]\). Compare.

Show carefully that for all \(x\) in \([\text{what interval?}]\), \(g_p(x) = f(2x)\). Use this to show how you can get the full Fourier series for the function \(g\) restricted to the interval \([0, \pi]\), as requested in for this problem. You will need to think carefully and explain the domains involved; the formula given a few lines above for \(g_p\) is valid on the interval \([-\pi/2, \pi/2]\); you want an expansion of the restriction of \(g\) on \([0, \pi]\). You will, of course, need to use periodicity properties of Fourier series.

4. (There is no Problem 4 for this part.)

The following problems focus on checking your answers, in the sense of checking some points to see if the series converge the way they should at the given points. Many of the incorrect answers given in homework could have been seen to be wrong by doing such checks.

For some of these checks, you may find it convenient to use the numerical series given to you in an earlier problem (Problem 11 in Section ???[14.4; the problem number in this section was an error].)

5. Check the Fourier series for \(g\) at the points \(\pm \pi\), \(\pm \pi/2\), and 0 (five points altogether). (Rhetorical question: Would your answer survive this check?) Be sure to explain why the value you get is the value you should get for the point in question, given the nature of the function \(g\).

6. Repeat Problem 5 just above for the sine series on \([0, \pi]\) at the points \(\pi\), \(\pi/2\), and 0. (Don’t just repeat what you did before, although the calculations may be very similar; you can use the result of a previous calculation as long as what you are doing is completely clear, but explain why your answer is the correct answer.)