It was pointed out in class on Friday, May 29, that there is a SIMILARITY between integrals and sums, and between integral notation and summation notation.

Consider, for example, the similarities between
\[ \int_1^5 f(x) \, dx \quad \text{and} \quad \sum_{k=1}^{5} a_k; \]
to make the similarity stronger, we could write the integral as \[ \int_{x=1}^{5} f(x) \, dx. \]
WE ARE NOT SAYING THAT THE INTEGRAL AND THE SUM ARE THE SAME THING, OR A SUM CAN BE “REPLACED” BY AN INTEGRAL, ARBITRARILY. You should understand both sums and integrals well enough that this is obvious; it is obvious that they are not the same.

For example, there is a similarity between \[ \int_1^5 2x \, dx \quad \text{and} \quad \sum_{k=1}^{5} 2k. \]
THESE ARE NOT THE SAME THING; you should understand sums and integrals well enough to see that they are not the same thing. We are not saying that
\[ \sum_{k=1}^{5} 2k = \int_{1}^{5} 2x \, dx \quad \text{(NOT)} \]

Some of the similarities:
Variables (“dummy” variables) can be changed without affecting the integral or the sum:
\[ \int_1^5 2x \, dx = \int_1^5 2y \, dy \quad \text{and} \quad \sum_{k=1}^{5} 2k = \sum_{j=1}^{5} 2j. \]
BUT THE SUM AND THE INTEGRAL ARE NOT THE SAME THING.
There is no EQUAL sign between the sum and the integral.

Similarly, we can change variables in sums and integrals as shown here:
\[ \sum_{k=1}^{5} 2k = \sum_{j=2}^{6} 2(j - 1) = \sum_{k=2}^{6} 2(k - 1) \quad \text{and} \quad \int_1^5 2x \, dx = \int_2^6 2(y - 1) \, dy = \int_2^6 2(x - 1) \, dx. \]

This idea of comparing integrals with sums was brought up in class in connection with comparing the idea of the sum of a constant sequence with the integral of a constant function:

Everyone knows (I hope) that one can integrate a constant function, even if there is no “x” explicitly in the formula for the function. For example,
\[ \int_1^5 20 \, dx \quad \text{makes complete sense; any first semester calculus student should understand it. One is integrating the constant function whose formula can be written } f(x) = 20. \]

Similarly,
\[ \sum_{k=1}^{5} 20 \quad \text{makes complete sense. One is adding the constant sequence whose formula can be written } a_k = 20. \]
\[ \sum_{k=1}^{5} 20 = 20 + 20 + 20 + 20 + 20 \quad \text{[5 times]} = 100. \]
But we are not saying that the sum can be replaced by the integral or that the sum equals the integral. We are COMPARING the integral with the sum.