Models of Sea Shells

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These notes give a brief explanation of the significance of the parameters used in the Sea_Shells MATLAB GUI
A sea shell is a **surface in 3-space**, which can be thought of as resulting from the motion of a *generating curve* along a *structural curve*. The latter describes the global shape of the shell while the former models the shape of the shell aperture.

These notes describe a **13-parameter model** of sea shells, based on [1]. The equations we use are a slight modification of those presented in [1], but the notation is similar.

The parameters values used for predefined shells in the MATLAB GUI Sea_Shells are taken from [1] and [2].

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The basic structure of a shell is defined by a curve in 3-space, the *structural curve*, shown in yellow on the figures.

Seen from above, this curve looks like a logarithmic spiral of equation

$$\rho = A \sin(\beta) \exp (\theta \cot(\alpha)),$$

where $A$, $\alpha$ and $\beta$ are parameters of the model.

The distance $R$ illustrated in the lower figure is given by

$$R = A \exp (\theta \cot(\alpha)).$$
A second curve, whose basic shape is an ellipse parametrized by $s$ (see figure), is used to generate the outer surface of the shell.

The parameters $a$ and $b$ are the half lengths of respectively the major and minor axes of the ellipse.

The ellipse is further rotated by and angle $\mu$ about its major axis, by an angle $\Omega$ about the vertical axis, and by an angle $\Phi$ about a vector normal to its plane.
For shells with “bumps”, five extra parameters are needed:

1. $P$ is an angle measuring the position of the bump along the ellipse;
2. $L$ measures the height of each bump;
3. $W_1$ measures the width of each bump along the ellipse;
4. $W_2$ measures the width of each bump along the logarithmic spiral;
5. $N$ is the number of bumps encountered as the angle $\theta$ is rotated by $2\pi$. 
Based on the above description, the parametric equations describing the shell surface are as follows.

\[
\begin{align*}
x &= \exp(\theta \cot(\alpha)) \left[ A \sin(\beta) \cos(\theta) + h(s, \theta) \right] \\
&\quad \left[ \cos(s + \Phi) \cos(\Omega + \theta) - \sin(s + \Phi) \sin(\mu) \sin(\theta + \Omega) \right] \\
y &= \exp(\theta \cot(\alpha)) \left[ -A \sin(\beta) \sin(\theta) - h(s, \theta) \right] \\
&\quad \left[ \cos(s + \Phi) \sin(\Omega + \theta) + \sin(s + \Phi) \sin(\mu) \cos(\theta + \Omega) \right] \\
z &= \left[ -A \cos(\beta) + h(s, \theta) \sin(s + \Phi) \cos(\mu) \right] \exp(\theta \cot(\alpha)),
\end{align*}
\]

\[
h(s, \theta) = \left( \left( \frac{\cos(s)}{a} \right)^2 + \left( \frac{\sin(s)}{b} \right)^2 \right)^{-1/2} \\
&\quad + L \exp \left( - \left( \frac{2(s - P)}{W1} \right)^2 - \left( \frac{2 l(\theta)}{W2} \right)^2 \right) \\
l(\theta) &= \frac{2\pi}{N} \left( \frac{N\theta}{2\pi} - \text{int} \left( \frac{N\theta}{2\pi} \right) \right).
\]