1. Find the derivatives of the following functions,
(a) \( f(x) = 2 \sin(x) \), \( f'(x) = (\ln 2) 2 \sin x \cos x \).
(b) \( f(s) = \tan(s^2) \), \( f'(s) = \sec^2(s^2) 2s \).
(c) \( g(z) = \frac{1}{\sqrt{1 + z^3}} \), \( g'(z) = -\frac{1}{2} (1 + z^4)^{-3/2} 4z^3 \).
(d) \( w(t) = \ln(\sin(t)) \), \( w'(t) = \frac{\cos t}{\sin t} = \cot t \).
(e) \( g(s) = \frac{2s}{(1-s^2)^{3/2}} \), \( g'(s) = \frac{(1-s^2)^{3/2} \frac{d}{ds}(2s) - 2s \frac{d}{ds}(1-s^2)^{1/2}}{(1-s^2)^{3/2}} \)
\( = \frac{2(1-s^2)^{3/2} + \frac{4}{3}s^2(1-s^2)^{-1/2}}{(1-s^2)^{3/2}} = \frac{2-2s^2}{(1-s^2)^{3/2}} \).
(f) \( g(x) = \sin \left( \frac{1+x^2}{1-x^2} \right) \), \( g'(x) = \cos \left( \frac{1+x^2}{1-x^2} \right) \frac{d}{dx} \left( \frac{1+x^2}{1-x^2} \right) = \cos \left( \frac{1+x^2}{1-x^2} \right) \frac{(1-x^2)+(1+x^2)}{(1-x^2)^2} = \frac{2}{(1-x^2)^2} \cos \left( \frac{1+x^2}{1-x^2} \right) \).
(g) \( h(s) = \arccos(\sin(s)) \), \( h'(s) = -\frac{1}{\sqrt{1-\sin^2 s}} \cos s = -1 \) for \( \cos(s) \geq 0 \) or \( -\frac{\pi}{2} \leq s \leq \frac{\pi}{2} \).

2. Consider the function \( f(x) = 2x^3 + 3x^2 - 36x + 1 \).
(a) Find the derivative, \( f' \), and draw its graph. Use this graph to find the interval(s) on which \( f(x) \) is increasing and the interval(s) on which \( f(x) \) is decreasing.

The derivative \( f'(x) = 6(x^2 + x - 6) = 6(x-2)(x+3) \) is positive for \( x < -3 \) and \( x > 2 \) so \( f(x) \) is an increasing function on these domains. \( f'(x) \) is negative for \( -3 < x < 2 \) so \( f(x) \) is decreasing on this interval.
(b) Compute \( f'' \) and use this to find the point at which the graph of \( f \) changes concavity.

\( f''(x) = 6(2x+1) \). This is negative for \( x < -\frac{1}{2} \) and positive for \( x > -\frac{1}{2} \). Thus \( f(x) \) is concave down in the first instance and concave up in the second.
(c) Determine if \( f(x) = 0 \) has 1 or 3 roots.
\( f(-3) > 0 \) and \( f(2) < 0 \) so \( f(x) \) has 3 roots.

3. Use the method of implicit differentiation to find \( \frac{dy}{dx} \) where,
\[ x + x^{-1} + y + y^{-1} = 4.5 \]

Find the equation of the tangent to this curve that passes through the point \( x = 1, y = 2 \).

\( \frac{dy}{dx} = \frac{-x^2-1}{x^2} \) so the slope at \( x = 1, y = 2 \) is 0 and the tangent line is the horizontal line \( y = 2 \).

4. A disk shaped oil slick from a tanker that has run aground increases in radius from 200 meters to 220 meters in 10 minutes. If the thickness of the slick is 1 millimeter find the (approximate) rate at which oil is leaking from the tanker when the radius of the oil slick is 200 meters. The answer should be in cubic meters per minute.

\( V = 0.001 \pi r^2 \) so \( \frac{dV}{dt} = 0.002 \pi r \frac{dr}{dt} \). At the time in question \( r = 200 \) and \( \frac{dr}{dt} \) is approximately \( \frac{220-200}{10} = 2 \).

5. The hyperbolic sine, \( \sinh \), and the hyperbolic cosine, \( \cosh \), are defined by the formulas,
\[ \sinh t = \frac{e^t - e^{-t}}{2}, \text{ and } \cosh t = \frac{e^t + e^{-t}}{2}. \]
The curve \( t \to (\cosh t, \sinh t) \) is a parametric representation of an hyperbola. Let \( m(t) \) be the slope of the curve (i.e., \( \frac{dy}{dx} \)) at time \( t \). Find the two limits,

\[
\lim_{t \to +\infty} m(t), \quad \text{and}, \quad \lim_{t \to -\infty} m(t).
\]

The slope \( m(t) = \frac{y'(t)}{x'(t)} = \frac{e^t + e^{-t}}{e^t - e^{-t}} \) and,

\[
\lim_{t \to \pm\infty} \frac{e^t + e^{-t}}{e^t - e^{-t}} = \pm 1.
\]