The following assignment is to be turned in on Thursday, February 11, 2016.

1. Three couples are invited to a dinner party. They will independently show up with probabilities 0.9, 0.8, and 0.75 respectively. Let $N$ be the number of couples that show up. Calculate the probability that $N = 2$.

2. Statistics show that 5% of men are color blind and 0.25% of women are color blind. If a person is randomly selected from a room with 35 men and 65 women, what is the likelihood that they are color blind?

3. You have two boxes. Box 1 contains 3 white balls and 4 black balls. Box 2 contains 2 white balls and 6 black balls. Here is an experiment. Pick a ball at random from Box 1 and put it into Box 2. Afterwards, pick a ball at random from Box 2. What is the probability that the ball you picked from Box 2 is black?

4. On a multiple choice exam with four choices for each question, a student either knows the answer to a question or marks it at random. Suppose the student knows the answers to 60% of the exam questions. If he marks the answer to question 1 correctly, what is the probability that he knows the answer to that question?

5. In a certain city, 30% of the people are conservative, 50% are liberals, and 20% are independents. In a given election, 2/3 of the conservatives voted, 80% of the liberals voted, and 50% of the independents voted. If we pick a voter at random, what is the probability that this person is a liberal?

6. Let $(\Omega, \mathcal{F}, P)$ be a probability space and suppose that $\{A_n\}_{n=1}^\infty$ is an increasing sequence of events. For each integer $n \geq 1$, set

$$C_n = \begin{cases} A_1 & \text{if } n = 1 \\ A_n \setminus A_{n-1} & \text{for } n \geq 2. \end{cases}$$

Show that the $C_n$’s are mutually disjoint and that

$$\bigcup_{n=1}^\infty A_n = \bigcup_{n=1}^\infty C_n.$$
7. Let \((\Omega, \mathcal{F}, P)\) be a probability space and suppose that \(\{A_n\}_{n=1}^\infty\) is a sequence of events. Set 
\[ B_n = \bigcup_{m=n}^\infty A_m \quad \text{and} \quad C_n = \bigcap_{m=n}^\infty A_m \]
It is clear that \(B_n\) is a decreasing sequence of events, while \(C_n\) is an increasing sequence of events. Show that 
\[ B = \bigcap_{n=1}^\infty B_n = \{\omega \in \Omega : \omega \in A_n \text{ for infinitely many values of } n\} \]
and 
\[ C = \bigcup_{n=1}^\infty C_n = \{\omega \in \Omega : \omega \in A_n \text{ for all but finitely many values of } n\} \]

8. Let \((\Omega, \mathcal{F}, P)\) be a probability space with 
\[ \Omega = \{1, 2, 3, 4, 5, 6\} \quad \text{and} \quad \mathcal{F} = \{\emptyset, \{2, 4, 6\}, \{1, 3, 5\}, \Omega\} . \]
Let \(U : \Omega \to \mathbb{R}\) be given by \(U(\omega) = \omega\).
Let \(V : \Omega \to \mathbb{R}\) be given by 
\[ V(\omega) = \begin{cases} 1 & \text{if } \omega \text{ is even}, \\ 0 & \text{if } \omega \text{ is odd}. \end{cases} \]
Let \(W : \Omega \to \mathbb{R}\) be given by \(W(\omega) = \omega^2\).
Explain whether or not these functions are discrete random variables.

9. Suppose we roll two fair 6-sided dice. Let \(X\) be a random variable corresponding to the minimum value of the two rolls. Find the probability mass function \(f_X\) corresponding to the random variable as a table of values (see below).

10. The probability mass function of a discrete random variable \(X\) is given below as a table of values. Compute the following:
   a) the probability that \(X\) is even (here we regard 0 and -4 as even)
   b) the probability that \(1 \leq X \leq 8\)
   c) the probability that \(X\) is -4 given that \(X \leq 0\)
   d) the probability that \(X \geq 3\) given that \(X > 0\)

\[
\begin{array}{cccccc}
 x & -4 & -1 & 0 & 2 & 4 & 5 & 6 \\
 f_X(x) & 0.15 & 0.2 & 0.1 & 0.1 & 0.2 & 0.2 & 0.05 \\
\end{array}
\]