HW 1 Math 464
Due in class Friday, Sep. 9, 2016.

1. Let $A, B, C$ be events in a probability space. Write each of the following event in terms of $A, B, C$ using intersections, unions and complements:
   (a) All of the events $A, B, C$ occur.
   (b) $A$ occurs and at least one of $B$ or $C$ occurs.
   (c) Exactly one of $A, B, C$ occurs.
   (d) Exactly two of $A, B, C$ occur.
   (e) At least one of $A, B, C$ occurs.
   (f) None of the events $A, B, C$ occur.

Now suppose that $P(A) = .5$, $P(B) = .6$, $P(C) = .5$, $P(A \cap B) = .3$, $P(A \cap C) = .2$, $P(B \cap C) = .3$ and $P(A \cap B \cap C) = .1$. Find the probabilities of the events (a) - (f) above.

2. Consider the probability space of tossing two fair, independent coins. Each coin has an $H$ side and a $T$ side. Define three events $A, B, C$ for which each pair of events is independent, but which the three events together $A, B, C$ are not independent.

3. On the real line $\mathbb{R}$, points $a$ and $b$ are selected at random such that $0 \leq a \leq 3$ and $-2 \leq b \leq 0$. This means that for instance the chance $a$ belongs to $(1/2, 2)$ equals $1.5/3$. Find the probability that the distance between $a$ and $b$ is greater than 3.

4. A pair of fair dice is thrown (the outcomes are equally likely). If the two numbers appearing are different, find the probability that (a) the sum is 6, (b) a ‘1’ appears on one of the tosses, (c) the sum is 4 or less.

5. A box contains 10 coins where 5 coins are two-headed, 3 coins are two-tailed, and 2 are fair coins. A coin is chosen at random and tossed. If a head appears, find the probability that the coin is fair.

6. Show that $A$ and $B$ are independent events if and only if $A$ and $B^c$ are independent events. [For proofs, must write out the details in a logical manner–pictures do not suffice].