MATH 456
Instructor V. E. Zakharov

Homework 1

1. Determine cosine Fourier series of

\[ f(x) = |\sin \frac{\pi n}{L} x| \quad 0 < x < L \]

2. Determine sine and cosine Fourier series of

\[ f(x) = \begin{cases} 
  x & 0 < x < L \\
  2L - x & L < x < 2L \\
  0 & 0 < x < 2L 
\end{cases} \]

3. Plot the function

\[ f(x) = \frac{4}{\pi} \sum_{k=0}^{4} \frac{(-1)^k}{k+1} \cos(k+1)x \quad -\pi < x < \pi \]

4. Proof that

\[ \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8} \]

Hint: Use Parseval’s equality and cosine Fourier series for the function

\[ f(x) = \begin{cases} 
  1 & -1/2 < x < 1/2 \\
  0 & |x| > 1/2 \\
  -1 & -1 < x < 1 
\end{cases} \]

5. Determine the complex Fourier series of

\[ f(x) = |\cos \frac{\pi n}{L} x| \quad -L < x < L \]

In problems 1,5 one can study the cases \( n = 1, 2 \) only. Those who will cope with a general case will get extra credits.
6. Prove that functions

\[ f_n(x) = \cos \frac{\pi x}{L} \left( n + \frac{1}{2} \right) \]

form an orthogonal system on the interval \( 0 < x < L \) and determine coefficient \( c_n \) in the expansion

\[ f(x) = L^2 - x^2 = \sum_{n=0}^{\infty} c_n f_n(x) \]

7. Find the Fourier sine series of

\[ f(x) = x^3 - 4x \quad 0 < x < 2 \]

Use Parseval’s equality to show that

\[ \sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi^6}{945} \]