Math 496T Spring 2023
Random Walks on Networks

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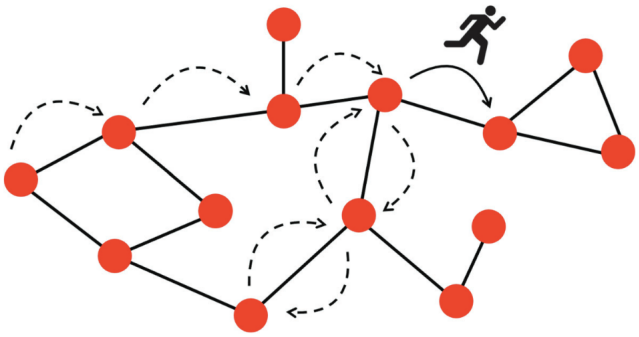

A gambler with infinitely-deep pockets playing a fair game will eventually break even. Wind instruments are possible in our 3-dimensional world, but they would not work in Flatland. A lost person is guaranteed to make it home (even with some road closures), but a confused bird may never return to its nest. These ideas can be formalized with the language of random walks on lattices. Around the turn of the twentieth century, Lord Rayleigh made a connection between such processes and classical electrical theory, a relationship which persists in some modern research in probability theory. In this topics course, we will study this connection as part of the basic theory of random walks on networks. While interesting in its own right, this theory is also the basis for many cutting-edge search and machine learning algorithms.

Prerequisites. Linear algebra at the level of Math 313, an introduction to probability at the level of MATH 464, and a willingness to engage with proofs. Anyone is welcome to discuss an alternative background with me.

Course Objectives. The aim of this course is to explore and appreciate the basics of the theory of random walks on graphs, especially the useful analogy with electrical network theory from physics. As a central feature, students will see a proof of Polya's famous theorem that a simple random walk on $\mathbb{Z}$ or $\mathbb{Z}^2$ is recurrent, while it is transient on $\mathbb{Z}^d$, $d \geq 3$. An additional goal is for students to develop and strengthen probabilistic reasoning and be exposed to important tools from the theory of Markov chains, martingales, and harmonic functions.

Text. P. G. Doyle and J. L. Snell, *Random Walks and Electric Networks*, 2006, Open source.

Figure: Davies et al., Network meta-analysis and random walks, *Statistics in Medicine*, 2022.