Convexity in the study of partial differential relations

Elementary calculus addresses the fundamental problem of analyzing a function in terms of locally affine (linear) objects. The converse problem of synthesizing a function from locally affine pieces is much harder.

A prototypical problem in this direction is to determine a function $u : \Omega \subseteq \mathbb{R}^n \to \mathbb{R}$ such that $W(\nabla u) = 0$ for a given function $W : \mathbb{R}^n \to \mathbb{R}$, or equivalently $\nabla u \in K$ for some subset $K \in \mathbb{R}^n$. Such problems arise in a variety of contexts including problems in topology, the theory of isometric immersions, optimal control theory and even material science. We will discuss some of the issues involved in the analysis of this problem, highlighting the role of convexity. We will also discuss some of the methods that are used in constructing solutions, including Gromov's theory of *convex integration*. Finally, we will look at a few of the many avenues for further research into this problem.