Introduction to the Theory of Nonlinear Waves

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In this course I will describe basic principles of the theory of nonlinear waves, including theory of weak turbulence.

Program

1. Hamiltonian dynamics of systems with finite number of degrees of freedom. Canonical transformation. Generating functions.
2. Hamiltonian dynamics in homogenous continuous media. Normal variavles. Waves of positive and negative energy. Classification of interactions. Examples.
3. Three-wave interactions. Second harmonic generation. Decay instability of monochromatic waves.
4. Canonical transformation in homogenous continuous media. Excluding of cubic terms in the Hamiltonian. Effective four-wave Hamiltonian. Decay instability of second order. Modulational instability.
5. Nonlinear Schrodinger equation and systems of such equations.
6. Solutions in NSLE and their stability.
7. Wave collapses. Theory of 3-d self-focusing.
8. Self-focusing in two dimensions.
9. Other examples of wave collapses.
10. Parametric excitations of nonlinear waves. S-theory.
11. Statistical description of nonlinear wave fields. Wyld's diagram technique.
12. Derivation of kinetic equations.
13. Kolmogorov solutions of three-wave kinetic equations. Capillary turbulence.
14. Kolmogorov spectra of four-wave kinetic equations. Self-similar solutions.
15. Theory of optical turbulence.
16. Theory of wind-driven sea.

Prerequisites: Math 252, Math 424 or 456.

Textbooks: there are no textbooks.
Some useful information is contained in the monographs:

1. Kolmogorov spectra of turbulence. By V.E. Zakharov, V.S. Lvov and G. Falkovich. Springer-Verlag, 1992.
2. Wave turbulence under parametric excitation. Springer series in nonlinear dynamics. 1994.
3. Nonlinear Schrodinger equations: self-focusing and wave collapses. By Catherine Sulem and Pierre-Louis Sulem. 1999.