MATHEMATICS 496/596: COMPUTABILITY AND LOGIC

A Fall 2012 topics course proposal

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Ever since the groundbreaking work of Gödel, mathematical logic was close to the theory of computation. Work of Turing, Church and others related the concepts of completeness and consistency of mathematical systems to those of computability of functions and decidability of formulae by algorithmic procedures. The two theories turned out to be so closely connected that it became possible to present the central ideas of Gödel in the language of Turing machines and recursive functions. In following years study of complexity of algorithms and of its theoretical limits gave a stimulus to further development of mathematical theory of computation, in particular in the work of Kolmogorov. In recent decades the study of the class P of problems solvable in polynomial time and the related class NP were studied extensively and further close relations between logic, proof theory and theory of algorithms were brought to light in the work of Valiant, Tarjan, Chaitin and Wigderson, to name a few of the most important mathematicians in the field. The $P \neq NP$ conjecture is one of the seven Millenium Problems. The new theory of quantum computing and quantum algorithms provided further motivation for research in the foundations of theory of algorithms, complexity and their relation to mathematical logic, following the work of Shor and others. The subject of computability and logic is central to both understanding the structure of mathematical theories and to efficient use of computers in mathematics and its applications.

The course will be an introduction to these fascinating themes. Its principal goal is to present the classical theorems of Gödel and Church, with complete proofs, in the context of computability theory, studying on the way Turing machines, recursive functions and also the relevant concepts in logic and model theory. All these tools will be introduced carefully. I will follow the general plan of “Computability and logic,, by G. Boolos et al., adding material from other sources as needed. An approximate plan of the course follows.

1. Countable sets and diagonalization
2. Computability and Turing machines
3. Recursive funstions
4. First-order logic
5. Model theory
6. Proof theory and completeness of propositional calculus
7. Gödel numbers and arithmetization of mathematical theories
8. Incompleteness and undecidability theorems of Gödel and Church
9. Gödel’s theorem about unprovability of consistency

If time permits, I will include an introduction to P and NP classes. No prior knowledge of mathematical logic or computation theory will be assumed and we will start from a detailed study of countable sets and diagonalization procedures. The only prerequisite is general mathematical culture. Every mathematics or applied mathematics graduate student certainly has the background to take the course. Motivated undergraduate students are welcome. I will be happy to discuss the course’s content with all those interested.