Math 518-001 – Lux

Special topics class Fall 2015: Algebras and their representations

The notion of an algebra over a field goes back to the beginning of the 20th century and generalizes the notion of a noncommutative ring. Algebras arise in several different areas for example group theory, number theory, and differential geometry. Examples range from matrix algebras, group algebras, endomorphism algebras, division algebras, to (quantum) universal enveloping algebras.

The most important way to study an algebra is via its homomorphisms into matrix algebras also called the (matrix)representations of the algebra. Representation theory of algebras turns out to be of essential use in such diverse topics as the representation theory of groups, the representation theory of Lie algebras, and the study of the Brauer group of a field in algebraic number theory.

In differential geometry the representations of Frobenius algebras play a central role in the classification of 2-dimensional topological quantum field theories and the representations of quiver algebras arise in string theory.

The goal of this special topics course is to give an introduction to the representation theory of algebras following mainly the Graduate Text in Mathematics, ”Associative Algebras” by Richard S. Pierce, published by Springer (the electronic version is available for free).

We will give an overview over the structure theory for algebras, in particular the theory of their indecomposable modules and their projective modules. Several classes of algebras will be studied in detail: Frobenius algebras, symmetric algebras, and division algebras. The notion of a quiver algebra will lead us to a proof of one of the Brauer-Thrall conjectures. Other topics that we will try to highlight is the (Hochschild) cohomology of algebras and the theory of Brauer groups.

This special topics course should be of interest to students with an interest in algebra, group theory, number theory, mathematical physics, and differential geometry. As a prerequisite the material covered in the first year core course in abstract algebra should suffice.