MATH 488-588

Solitons in Mathematics and Physics

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Course Content

Solitons, the nonlinear localized objects, play a very important role in different areas: nonlinear optics, hydrodynamics, plasma theory, superfluidity, and magnetism. Also, they are important for the theory of general relativity – the black holes are solitons. It is remarkable that such a broad variety of physical phenomena, from microscopic to astronomic scale, can be described by unified mathematical apparatus that was intensively developed during last four decades. The mathematical theory of solitons, known as the Inverse scattering method, is closely connected to the spectral theory of differential operators and to the classical theory of integrable Hamiltonian systems.

In this course we will discuss the basic elements of both physics and mathematics of solitons. We will make accent on pure algebraic method for construction of solitonic solutions via local and non-local d-bar problem. Then we will develop the Inverse scattering method for the Schrodinger and Dirac operators. The course will be organized as follow:

1. Basic integrable models in nonlinear wave dynamics and their interconnection. Lax representations. Gauge equivalence. Simple solitonic solutions of basic equations: KdV, NLSE, N-wave, KP-1, KP-2, sine-Gordon equations. (3 weeks)
2. Method of matrix Riemann-Hilbert problem in 1+1 dimensions. Construction of multi-solitonic solutions. Interaction of solitons. Breathers. (3 weeks)
3. Method of non-local d-bar problem (dressing method). Solitonic solutions of KP-1 and KP-2 equations. Solutions depending on functional parameters. Breathers. (3 weeks)
4. Method of Inverse scattering transform for KdV and NLS equations. Symmetry groups of integrable systems. (3 weeks)
5. Solitons in Optic fibers. (1 week)
6. Solitons in Hydrodynamics. Rogue waves. (1 week)
7. Solitons in General Relativity. Black holes. (1 week).

The course is intended for graduate students but will also be available for daring undergraduates with basic knowledge of ODE, linear algebra and complex analysis: prerequisites MATH 215, 254, 322 or 422. Lecture notes will be posted.