Monte Carlo Methods

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Description: Monte Carlo (MC) methods are numerical algorithms for generating samples from probability distributions. They are widely used in scientific, engineering, and statistical computing, and are especially applicable to problems in statistical mechanics, Bayesian inference, operations research, chemical kinetics, and population dynamics, among many others.

This introductory course is aimed at graduate students in the mathematics, statistics, computer science, engineering, physical sciences, quantitative biology, or really any field where Monte Carlo methods are used. The course aims to strike a balance between practical implementation and application of various Monte Carlo methods on the one hand, and their mathematical analysis on the other.

Topics to be covered include:

- Brief review of probability theory
- Direct sampling methods
- Markov chains & Markov chain Monte Carlo (MCMC)
- Basic MC error analysis
- Variance reduction
- Importance sampling

Examples will be drawn from statistical mechanics, chemistry, Bayesian statistics, and other fields, depending in part on student interest. However, no background in these areas is assumed. Time permitting, more specialized topics may be covered. Possibilities include (but are not limited to)

- Sequential Monte Carlo / filtering
- Rare event simulations
- Gillespie & related algorithms
- Stochastic differential equations
- Exact sampling

Prerequisites: students should know probability at the undergraduate-level, e.g., Math 464 or equivalent, as well as linear algebra (e.g., Math 410 or equivalent). Some of this material will be briefly reviewed, and additional mathematical tools will be covered as needed. Interested students without this background are encouraged to see the instructor prior to registering for the course.

In addition to the mathematical prerequisites, the ability (or willingness to learn) to program in a suitable language such as Matlab, Python, R, Java, Julia, etc., is assumed. There will be programming assignments in this course.

Grading: grading will be based on a few problem sets and a semester project.

Textbook: none required. I plan to follow a variety of sources, some of which are listed below. I will also hand out lecture notes as needed.

General MC references:

1. J M Handscomb and D C Hammersley, _Monte Carlo Methods_, Methuen 1965

2. M H Kalos and P A Whitlock, _Monte Carlo Methods_, Wiley 2008

3. J S Liu, _Monte Carlo Strategies in Scientific Computing_, Springer 2008

4. A B Owen, _Monte Carlo theory, methods and examples_, 2013

More specialized references:

5. A Asmussen and P W Glynn, _Stochastic Simulation: Algorithms and Analysis_, Springer 2007

6. A D Sokal, "Monte Carlo methods in statistical mechanics: foundations and new algorithms," Functional Integration (Carg\'ese, 1996), 131–192, NATO Adv. Sci. Inst. Ser. B Phys., 361, Plenum, New York, 1997

Useful reference for the probability we need:

7. A J Chorin and O H Hald, _Stochastic Tools in Mathematics and Science_, Springer 2009