Symplectic geometry and integrable systems

Integrable systems is one of the most eclectic directions in modern mathematics, so the course will touch upon a variety of different areas, including:

- Symplectic and Poisson geometry;
- Lie groups and Lie algebras;
- Algebraic curves and their Jacobians (time permitted).

Tentative list of topics:

- Billiards, integrability of the billiard in an ellipse (1-2 weeks).
- Symplectic manifolds, Hamiltonian dynamics, Arnold-Liouville theorem (2 weeks).
- Spherical pendulum, integrable geodesic flows (1 week).
- Poisson manifolds, Lie groups and Lie algebras (2 weeks).
- The free rigid body and the spinning top (2 weeks).
- Toda lattice, Lax representation, r-matrix (2 weeks).
- Bi-Hamiltonian systems, multidimensional rigid body (1 week).

Additional topics (time permitted, possibly presented by students) include:

- Free rigid body and elliptic functions (1 week).
- Algebraic curves and their Jacobians. Linearization of integrable systems (2 weeks).
- Infinite-dimensional integrable systems, the KdV equation (1-2 weeks).
- Poisson-Lie groups and the pentagram map (1-2 weeks).

Text: The coverage of this course is very broad and there exist no good introductory texts covering all these topics. References include:

- S. Tabachnikov, "Geometry and billiards".
- M. Audin, "Spinning tops".

Lecture notes will be provided.

Prerequisites: Math 534 (Geometry and topology).

Expected learning outcomes:

- Be able to describe main concepts of symplectic geometry and their relevance in physics.
- Be able to provide several examples of integrable systems and describe the associated geometric/algebraic structures.
- Be able to present these ideas in a clear way with careful definitions and some proofs.